

Unit II – Divide and Conquer

- Merge sort
- **Quick sort**
- Binary search
- Multiplication of large Integers
- Strassen's Matrix Multiplication

QUICK SORT

- Partition the array such that all elements before the position are smaller than or equal to $A[s]$ and all the elements after the position s are greater than or equal to $A[s]$

$$\underbrace{A[0] \dots A[s-1]}_{\text{all are } \leq A[s]} \quad A[s] \quad \underbrace{A[s+1] \dots A[n-1]}_{\text{all are } \geq A[s]}$$

- Divide the array based on element selection – pivot
- Pivot selection can be done as follows
 - First element as pivot
 - Last element as pivot
 - Middle element as pivot
 - Random number as pivot

- Example

Quick Sort

1st element as pivot

0	1	2	3	4	5	6	7
5	3	1	9	8	2	4	7
P	i						j

5	3	1	9	8	2	4	7
			i			j	

5	3	1	4	8	2	9	7
			i			j	

5	3	1	4	8	2	9	7
			i		j		

5	3	1	4	2	8	9	7
			i		j		

5	3	1	4	2	8	9	7
				j	i		

$i \rightarrow 7, 5$

$j \rightarrow 6, 5$

Swap 4, 9

Swap 2, 8

$(i < j) \Rightarrow$ Not True

Exchange j & pivot

2	3	1	4	5	8	9	7
< 5					≥ 5		
Sub-array					Sub-array		

QUICK SORT ALGORITHM

Quicksort (l,h)

```
{  
  if ( l < h )  
  {  
    j= partition (l,h);  
    Quicksort (l,j);  
    Quicksort (j+1,h);  
  }  
}
```

Partition (l,h)

```
{  
  pivot=A[l];  
  i=l+1;  
  j=h;  
  while(i<j)  
  {  do  
    { i++; } while(A[i]<=pivot);  
    do  
    { j--; } while(A[j]>pivot);  
    if i<j  
      swap(A[i],A[j]);  
  } swap(A[l], A[j]);  
  return j;  
}
```

QUICK SORT ANALYSIS

- $n/2 / 2 / 2 \rightarrow n/2^k$
- $n/2^k = 1$
- $n = 2^k$
- $K = \log_2 n \rightarrow$ No. of levels
- Time for partitioning $\rightarrow n$
- So, the time complexity is **$O(n \log_2 n)$**
- Best case - $O(n \log_2 n)$, Average case - $O(n \log_2 n)$
- Worst case – sorted array , always partitioning happens at the beginning $O(n^2)$
- So to overcome the worst case of quick sort always select ***middle*** element as pivot and select a ***random*** element as pivot.

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Binary Search

- Efficient searching method

Binary Search

Search 23

0	1	2	3	4	5	6	7	8	9
2	5	8	12	16	23	38	56	72	91

23 > 16
take 2nd half

L=0 M=4 H=9

0	1	2	3	4	5	6	7	8	9
2	5	8	12	16	23	38	56	72	91

23 > 56
take 1st half

L=5 M=7 H=9

0	1	2	3	4	5	6	7	8	9
2	5	8	12	16	23	38	56	72	91

Found 23,
Return 5

L=5, M=5 H=6

0	1	2	3	4	5	6	7	8	9
2	5	8	12	16	23	38	56	72	91



Binary Search – Algorithm

BinarySearch($A[0..n-1], K$)

$l \leftarrow 0; r \leftarrow n-1$

while $l \leq r$ do

$m \leftarrow (l+r)/2$

 if $k = A[m]$ return m

 else if $k < A[m]$ $r \leftarrow m-1$

 else $l \leftarrow m+1$

return -1

Analysis

Worst case - $O(\log_2 n)$

Best case - $O(1)$

Average case - $O(\log_2 n)$

Binary Search – Analysis

- **Analysis**

$$C(n) = C(n/2) + 1$$

Sublist + middle element

Assume $n = 2^k$, $k = \log_2 n$

$$\begin{aligned} C(2^k) &= C(2^k/2^1) + 1 \\ &= C(2^{k-1}) + 1 \end{aligned}$$

Using Backward substitution

$$C(2^k) = [C(2^{k-2}) + 1] + 1 \quad C(2^{k-1}) = C(2^{k-2}) + 1$$

$$C(2^k) = C(2^{k-2}) + 2$$

$$C(2^k) = C(2^{k-3}) + 3$$

$$C(2^k) = C(2^{k-4}) + 4$$

$$C(2^k) = C(2^{k-k}) + k$$

$$C(2^k) = C(2^{k-k}) + k$$

$$C(2^k) = C(2^0) + k \rightarrow C(1) + \log_2 n \rightarrow 1 + \log_2 n \rightarrow \log_2 n$$