

UNIT 4

BOILING, CONDENSATION AND HEAT EXCHANGER



CONTENTS

- Boiling and condensation
- Regimes of boiling
- Forced convection boiling



BOILING

- Occurs at the solid–liquid interface when a liquid is brought into contact with a surface maintained at a temperature T_s sufficiently above the saturation temperature T_{sat} of the liquid
- By Newton's law of cooling,

$$q_{\text{boiling}} = h(T_s - T_{\text{sat}}) = h\Delta T_{\text{excess}} \text{ (W/m}^2 \text{)}$$

where, $\Delta T_{\text{excess}} = (T_s - T_{\text{sat}})$ -----> Excess temperature

TYPES OF BOILING

(1) Pool boiling – absence of bulk fluid flow

(2) Flow boiling (or forced convection boiling) – presence of bulk fluid flow

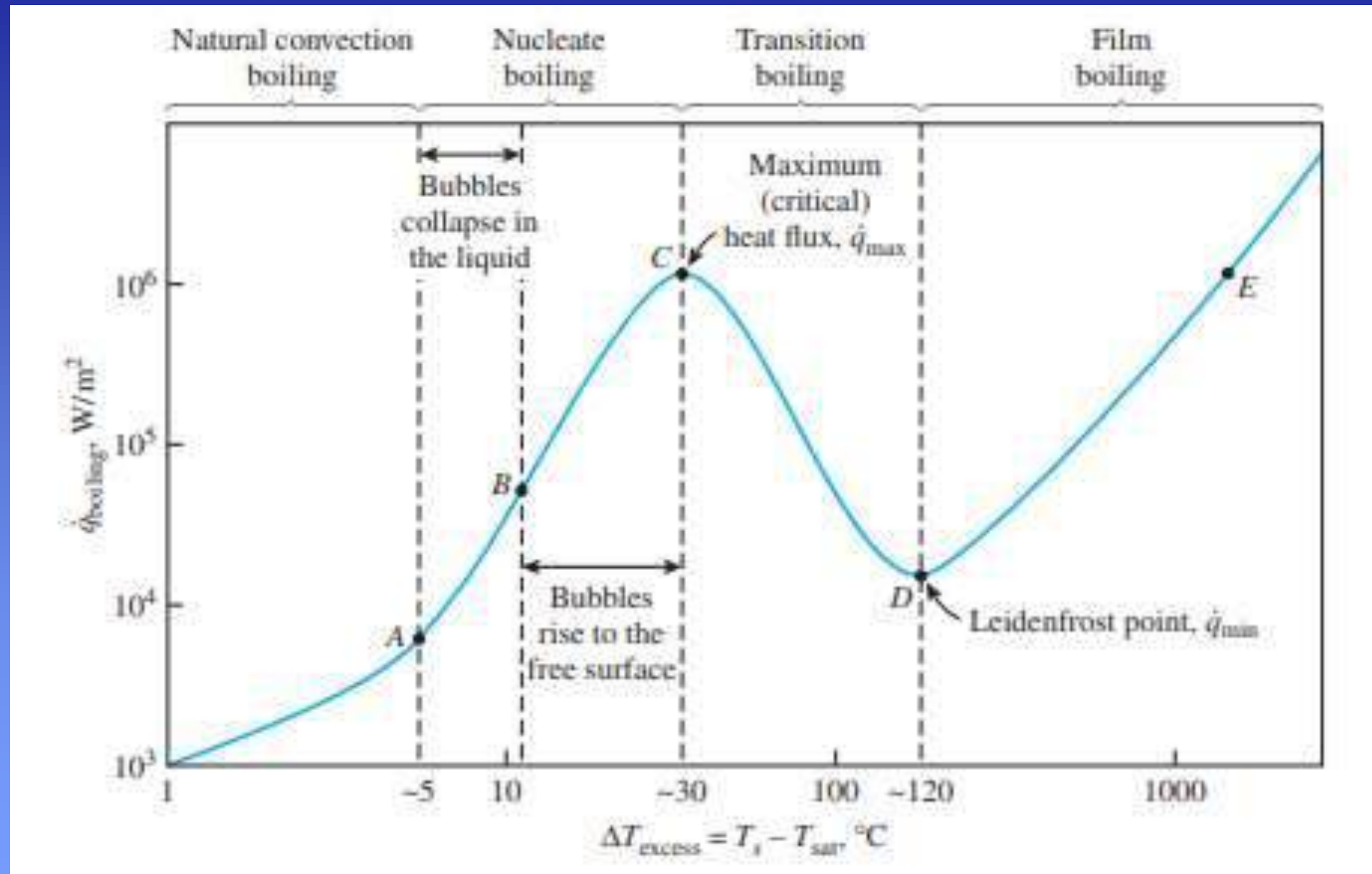


POOL BOILING

- The fluid is not forced to flow by a mover such as a pump, and any motion of the fluid is due to natural convection currents and the motion of the bubbles under the influence of buoyancy.



BOILING REGIMES AND THE BOILING CURVE

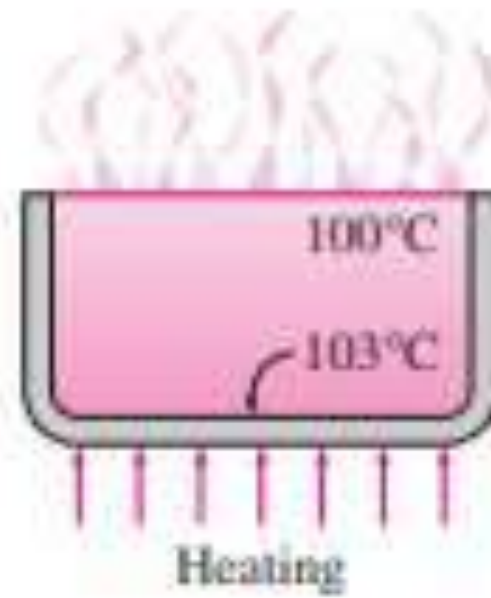


(a) Natural Convection Boiling (to Point A on the Boiling Curve)

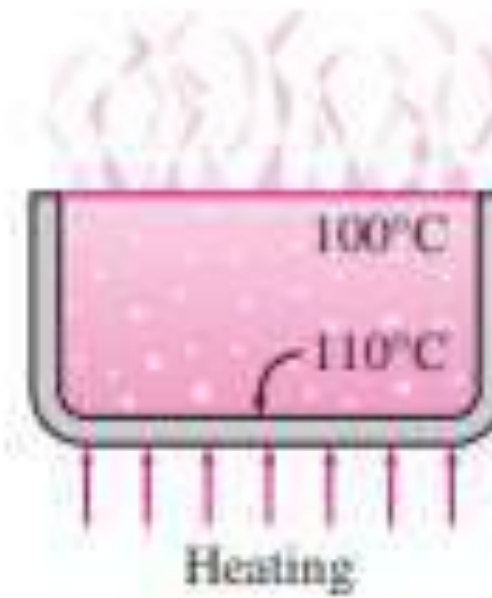
The liquid is slightly superheated in this case (a metastable condition) and evaporates when it rises to the free surface. The fluid motion in this mode of boiling is governed by natural convection currents, and heat transfer from the heating surface to the fluid is by natural convection

(b) Nucleate Boiling (between Points A and C)

Nucleate boiling is the most desirable boiling regime in practice because high heat transfer rates can be achieved in this regime with relatively small values of ΔT_{excess} , typically under 30°C for water



(a) Natural convection boiling



(b) Nucleate boiling



(c) Transition boiling



(d) Film boiling

(c) Transition Boiling (between Points C and D)

In the transition boiling regime, both nucleate and film boiling partially occur. Nucleate boiling at point C is completely replaced by film boiling at point D. Operation in the transition boiling regime, which is also called the unstable film boiling regime, is avoided in practice.

(d) Film Boiling (beyond Point D)

In this region the heater surface is completely covered by a continuous stable vapor film. Point D, where the heat flux reaches a minimum, is called the Leidenfrost point

HEAT TRANSFER CORRELATIONS IN POOL BOILING

(i) Nucleate boiling regime ($5^{\circ}\text{C} \leq \Delta T_{\text{excess}} \leq 30^{\circ}\text{C}$)

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n} \right]^3$$

where

$\dot{q}_{\text{nucleate}}$ = nucleate boiling heat flux, W/m^2

μ_l = viscosity of the liquid, $\text{kg/m}\cdot\text{s}$

h_{fg} = enthalpy of vaporization, J/kg

g = gravitational acceleration, m/s^2

ρ_l = density of the liquid, kg/m^3

ρ_v = density of the vapor, kg/m^3

σ = surface tension of liquid–vapor interface, N/m

c_{pl} = specific heat of the liquid, $\text{J/kg}\cdot^{\circ}\text{C}$

T_s = surface temperature of the heater, $^{\circ}\text{C}$

T_{sat} = saturation temperature of the fluid, $^{\circ}\text{C}$

C_{sf} = experimental constant that depends on surface–fluid combination

Pr_l = Prandtl number of the liquid

n = experimental constant that depends on the fluid

(ii) Peak Heat Flux

The maximum (or critical) heat flux in nucleate pool boiling was determined by

$$\dot{q}_{\max} = C_{cr} h_{fg} [\sigma g \rho^2 v (\rho_l - \rho_v)]^{1/4}$$

where C_{cr} is a constant whose value depends on the heater geometry

(iii) Minimum Heat Flux

The minimum heat flux for a large horizontal plate,

$$\dot{q}_{\min} = 0.09 \rho_v h_{fg} \left[\frac{\sigma g (\rho_l - \rho_v)}{(\rho_l + \rho_v)^2} \right]^{1/4}$$

(iv) Film Boiling

The heat flux for film boiling on a horizontal cylinder or sphere of diameter D is given by,

$$\dot{q}_{\text{film}} = C_{\text{film}} \left[\frac{g k_v^3 \rho_v (\rho_l - \rho_v) [h_{fg} + 0.4 c_{pv} (T_s - T_{\text{sat}})]}{\mu_v D (T_s - T_{\text{sat}})} \right]^{1/4} (T_s - T_{\text{sat}})$$

where k_v is the thermal conductivity of the vapor in W/mK and

$$C_{\text{film}} = \begin{cases} 0.62 & \text{for horizontal cylinders} \\ 0.67 & \text{for spheres} \end{cases}$$

Treating the vapor film as a transparent medium sandwiched between two large parallel plates and approximating the liquid as a blackbody, radiation heat transfer can be determined from

$$\dot{q}_{\text{rad}} = \epsilon \sigma (T_s^4 - T_{\text{sat}}^4)$$

For $q_{\text{rad}} < q_{\text{film}}$

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4} \dot{q}_{\text{rad}}$$

PROBLEMS

1.

Water is to be boiled at atmospheric pressure in a mechanically polished stainless steel pan placed on top of a heating unit, as shown in Fig. 10–15. The inner surface of the bottom of the pan is maintained at 108°C . If the diameter of the bottom of the pan is 30 cm, determine (a) the rate of heat transfer to the water and (b) the rate of evaporation of water.

The properties of water at the saturation temperature of 100°C are $\sigma = 0.0589 \text{ N/m}$ and

$$\rho_l = 957.9 \text{ kg/m}^3$$

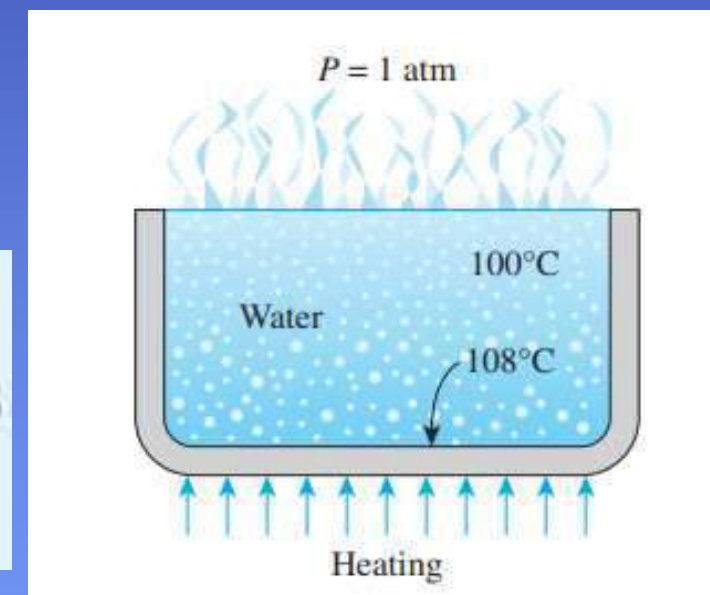
$$h_{fg} = 2257 \times 10^3 \text{ J/kg}$$

$$\rho_v = 0.6 \text{ kg/m}^3$$

$$\mu_l = 0.282 \times 10^{-3} \text{ kg/m}\cdot\text{s}$$

$$\text{Pr}_l = 1.75$$

$$c_{pl} = 4217 \text{ J/kg}\cdot\text{K}$$



for the boiling of water on a mechanically polished stainless steel surface

$$C_{sf} = 0.0130 \text{ and } n = 1.0$$

(a) The excess temperature in this case $\Delta T_{\text{excess}} = (T_s - T_{\text{sat}}) = 108 - 100 = 8^\circ\text{C}$ which is relatively low (less than 30°C).

$$\begin{aligned}\dot{q}_{\text{nucleate}} &= \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n} \right]^3 \\ &= (0.282 \times 10^{-3})(2257 \times 10^3) \left[\frac{9.81 \times (957.9 - 0.6)}{0.0589} \right]^{1/2} \\ &\quad \times \left(\frac{4217(108 - 100)}{0.0130(2257 \times 10^3)1.75} \right)^3 \\ &= 7.21 \times 10^4 \text{ W/m}^2\end{aligned}$$

The surface area of the bottom of the pan is

$$A = \pi D^2/4 = \pi(0.3 \text{ m})^2/4 = 0.07069 \text{ m}^2$$

Then the rate of heat transfer during nucleate boiling becomes

$$\dot{Q}_{\text{boiling}} = A \dot{q}_{\text{nucleate}} = (0.07069 \text{ m}^2)(7.21 \times 10^4 \text{ W/m}^2) = \mathbf{5097 \text{ W}}$$

(b) The rate of evaporation of water is determined from

$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{5097 \text{ J/s}}{2257 \times 10^3 \text{ J/kg}} = 2.26 \times 10^{-3} \text{ kg/s}$$

That is, water in the pan will boil at a rate of more than 2 grams per second.

2.

Water in a tank is to be boiled at sea level by a 1-cm-diameter nickel plated steel heating element equipped with electrical resistance wires inside, as shown in Fig. 10–16. Determine the maximum heat flux that can be attained in the nucleate boiling regime and the surface temperature of the heater in that case.

The properties of water at the saturation temperature of 100°C are $\sigma = 0.0589 \text{ N/m}$ and

$$\rho_l = 957.9 \text{ kg/m}^3$$

$$h_{fg} = 2257 \times 10^3 \text{ J/kg}$$

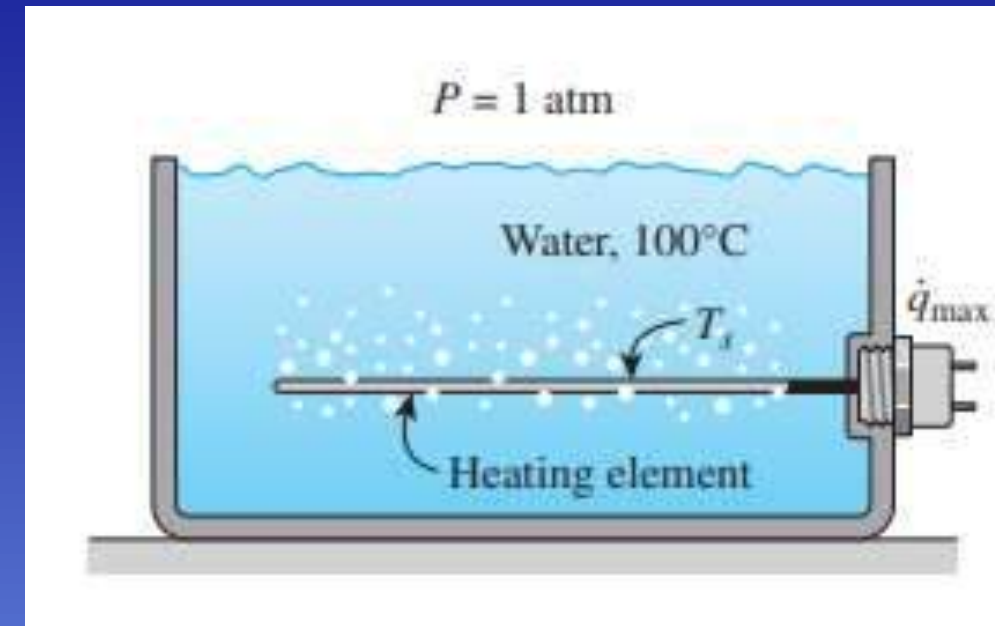
$$\rho_v = 0.6 \text{ kg/m}^3$$

$$\mu_l = 0.282 \times 10^{-3} \text{ kg/m}\cdot\text{s}$$

$$\text{Pr}_l = 1.75$$

$$c_{pl} = 4217 \text{ J/kg}\cdot\text{K}$$

For the boiling of water on a nickel plated surface



$$C_{sf} = 0.0060 \text{ and } n = 1.0$$

The heating element in this case can be considered to be a short cylinder whose characteristic dimension is its radius. That is, $L = r = 0.005 \text{ m}$. The dimensionless parameter L^* and the constant C_{cr} are determined from

$$L^* = L \left(\frac{g(\rho_l - \rho_v)}{\sigma} \right)^{1/2} = (0.005) \left(\frac{(9.81)(957.9 - 0.6)}{0.0589} \right)^{1/2} = 2.00 > 1.2$$

which corresponds to $C_{cr} = 0.12$.

Then the maximum or critical heat flux is determined from Eq. 10-3 to be

$$\begin{aligned} \dot{q}_{\max} &= C_{cr} h_{fg} [\sigma g \rho_v^2 (\rho_l - \rho_v)]^{1/4} \\ &= 0.12(2257 \times 10^3) [0.0589 \times 9.81 \times (0.6)^2 (957.9 - 0.6)]^{1/4} \\ &= \mathbf{1.017 \times 10^6 \text{ W/m}^2} \end{aligned}$$

The nucleate boiling heat flux for a specified surface temperature, can also be used to determine the surface temperature when the heat flux is given

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n} \right]^3$$

$$1.017 \times 10^6 = (0.282 \times 10^{-3})(2257 \times 10^3) \left[\frac{9.81(957.9 - 0.6)}{0.0589} \right]^{1/2} \\ \times \left[\frac{4217(T_s - 100)}{0.0130(2257 \times 10^3) 1.75} \right]^3$$

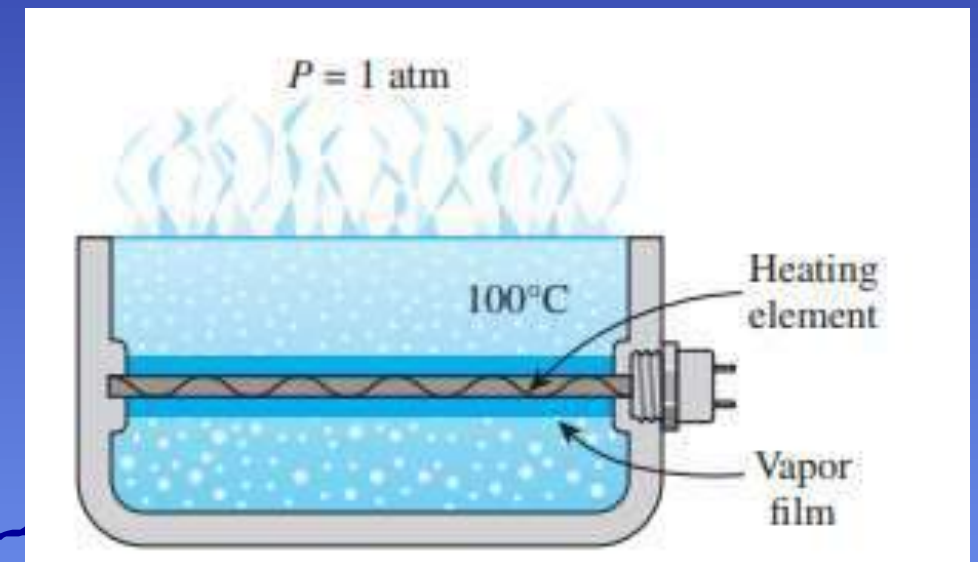
$$T_s = 119^\circ\text{C}$$

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3.

Water is boiled at atmospheric pressure by a horizontal polished copper heating element of diameter $D = 5 \text{ mm}$ and emissivity $\varepsilon = 0.05$ immersed in water, as shown in Fig. 10–17. If the surface temperature of the heating wire is 350°C , determine the rate of heat transfer from the wire to the water per unit length of the wire.

The properties of water at the saturation temperature of 100°C are $h_{fg} = 2257 \times 10^3 \text{ J/kg}$ and $\rho_f = 957.9 \text{ kg/m}^3$. The properties of vapor at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2 = (100 + 350)/2 = 225^\circ\text{C}$ are



$$\rho_v = 0.444 \text{ kg/m}^3$$

$$\mu_v = 1.75 \times 10^{-5} \text{ kg/m}\cdot\text{s}$$

$$c_{pv} = 1951 \text{ J/kg}\cdot\text{K}$$

$$k_v = 0.0358 \text{ W/m}\cdot\text{K}$$

The excess temperature in this case is $\Delta T_{\text{excess}} = (T_s - T_{\text{sat}}) = 350 - 100 = 250^\circ\text{C}$ which is much larger than 30°C for water. Therefore, film boiling will occur. The film boiling heat flux in this case can be determined from

$$\begin{aligned}\dot{q}_{\text{film}} &= 0.62 \left[\frac{g k_v^3 \rho_v (\rho_l - \rho_v) [h_{fg} + 0.4 c_{pv} (T_s - T_{\text{sat}})]}{\mu_v D (T_s - T_{\text{sat}})} \right]^{1/4} (T_s - T_{\text{sat}}) \\ &= 0.62 \left[\frac{9.81 (0.0358)^3 (0.444) (957.9 - 0.441)}{(1.75 \times 10^{-5}) (5 \times 10^{-3}) (250)} \right]^{1/4} \times 250 \\ &= 5.93 \times 10^4 \text{ W/m}^2\end{aligned}$$

The radiation heat flux is determined from

$$\begin{aligned}\dot{q}_{\text{rad}} &= \varepsilon \sigma (T_s^4 - T_{\text{sat}}^4) \\ &= (0.05)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(350 + 273 \text{ K})^4 - (100 + 273 \text{ K})^4] \\ &= 372 \text{ W/m}^2\end{aligned}$$

Note that heat transfer by radiation is negligible in this case because of the low emissivity of the surface and the relatively low surface temperature of the heating element. Then the total heat flux becomes

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4} \dot{q}_{\text{rad}} = 5.93 \times 10^4 + \frac{3}{4} \times 372 = 5.96 \times 10^4 \text{ W/m}^2$$

Finally, the rate of heat transfer from the heating element to the water is determined by multiplying the heat flux by the heat transfer surface area,

$$\begin{aligned}\dot{Q}_{\text{total}} &= A\dot{q}_{\text{total}} = (\pi DL)\dot{q}_{\text{total}} \\ &= (\pi \times 0.005 \text{ m} \times 1 \text{ m})(5.96 \times 10^4 \text{ W/m}^2) \\ &= \mathbf{936 \text{ W}}\end{aligned}$$



Table1: Values of C_{sl} for pool boiling

<i>S.No.</i>	<i>Liquid-surface</i>	<i>C_{sl}</i>
1.	Water - copper	0.013
2.	Water - brass	0.060
3.	Water - platinum	0.013
4.	Water - ground and polished stainless steel	0.008
5.	Water - mechanically polished stainless steel	0.013
6.	Benzene - chromium	0.010
7.	Ethanol - chromium	0.0027
8.	n-pentane - chromium	0.0150
9.	n-butanol - copper	0.003
10.	Isopropyl alcohol - copper	0.00225

4.

A wire of 1.2 mm diameter and 200 mm length is submerged horizontally in water at 7 bar. The wire carries a current of 135 A with an applied voltage of 2.18 V. If the surface of the wire is maintained at 200°C, calculate :

- (i) The heat flux, and
- (ii) The boiling heat transfer coefficient.

Solution. Given : $d = 1.2 \text{ mm} = 0.0012 \text{ m}$, $l = 200 \text{ mm} = 0.2 \text{ m}$, $I = 135 \text{ A}$, $V = 2.18 \text{ V}$, $t_s = 200^\circ\text{C}$.

(i) **The heat flux, q :**

The electrical energy input to the wire is given by

$$Q = VI = 2.18 \times 135 = 294.3 \text{ W}$$

Surface area of the wire, $A = \pi dl$

$$= \pi \times 0.0012 \times 0.2 = 7.54 \times 10^{-4} \text{ m}^2$$

$$\begin{aligned} \therefore q &= \frac{Q}{A} = \frac{294.3}{7.54 \times 10^{-4}} \\ &= 0.39 \times 10^6 \text{ W/m}^2 = \mathbf{0.39 \text{ MW/m}^2 \text{ (Ans.)}} \end{aligned}$$

(ii) **The boiling heat transfer coefficient, h :**

Corresponding to 7 bar, $t_{sat} = 164.97^\circ\text{C}$, and

$$q = h(t_s - t_{sat})$$

$$\text{or, } h = \frac{q}{(t_s - t_{sat})} = \frac{0.39 \times 10^6}{(200 - 164.97)}$$

$$= \mathbf{11133.3 \text{ W/m}^2\text{C} \text{ (Ans.)}}$$

5.

An electric wire of 1.25 mm diameter and 250 mm long is laid horizontally and submerged in water at atmospheric pressure. The wire has an applied voltage of 18 V and carries a current of 45 amperes. Calculate:

- (i) The heat flux, and
- (ii) The excess temperature.

The following correlation for water boiling on horizontal submerged surface holds good:

$$h = 1.58 \left(\frac{Q}{A} \right)^{0.75} = 5.62 (\Delta t_e)^3, \text{ W / m}^2 \text{ } ^\circ\text{C}$$

Solution. Given : $d = 1.25 \text{ mm} = 0.00125 \text{ m}$, $l = 250 \text{ mm} = 0.25 \text{ m}$, $V = 18 \text{ V}$, $I = 45 \text{ A}$.

(i) **The heat flux, q :**

Electrical energy input to the wire, $Q = VI = 18 \times 45 = 810 \text{ W}$

Surface area of the wire, $A_s = \pi dl = \pi \times 0.00125 \times 0.25 = 9.817 \times 10^{-4} \text{ m}^2$

$$\therefore q = \frac{Q}{A} = \frac{810}{9.817 \times 10^{-4}} = 0.825 \times 10^6 \text{ W/m}^2 = \mathbf{0.825 \text{ MW/m}^2} \text{ (Ans.)}$$

(ii) **The excess temperature, Δt_e :**

Using the correlation,

$$1.58 \left(\frac{Q}{A} \right)^{0.75} = 5.62 (\Delta t_e)^3 \quad \dots \text{given}$$

$$\text{or, } 1.58 (0.825 \times 10^6)^{0.75} = 5.62 (\Delta t_e)^3$$

$$\Delta t_e = \left[\frac{1.58 (0.825 \times 10^6)^{0.75}}{5.62} \right]^{0.333} = 19.68^\circ\text{C} \text{ (Ans.)}$$

5.

A nickel wire of 1 mm diameter and 400 mm long, carrying current, is submerged in a water bath which is open to atmospheric pressure. Calculate the voltage at the burnout point if at this point the wire carries a current of 190 A.

Solution. Given : $d = 1 \text{ mm} = 0.001 \text{ m}$; $l = 400 \text{ mm} = 0.4 \text{ m}$, $I = 190 \text{ A}$

The thermo-physical properties of water and vapour at 100°C are:

$\rho_l = (\rho_f) = 958.4 \text{ kg/m}^3$, $\rho_v = 0.5955 \text{ kg/m}^3$, $h_{fg} = 2257 \text{ kJ/kg}$, $\sigma = 58.9 \times 10^{-3} \text{ N/m}$.

Voltage at the burnout point, V_b :

At burnout *i.e.*, the points of critical heat flux, the correlation is

$$\begin{aligned} q_{sc} &= 0.18(\rho_v)^{1/2} h_{fg} [g\sigma(\rho_l - \rho_v)]^{1/4} \\ &= 0.18 (0.5955)^{1/2} \times 2257 \times 10^3 [9.81 \times 58.9 \times 10^{-3} (958.4 - 0.5955)]^{1/4} \\ &= 1.52 \times 10^6 \text{ W/m}^2 = 1.52 \text{ MW/m}^2 \end{aligned}$$

Electric energy input to the wire,

$$Q = V_b \times I$$

or,
$$q = \frac{Q}{A} = \frac{V_b \times I}{A} = q_{sc}$$

or,
$$V_b = \frac{A \times q_{sc}}{I} = \frac{\pi dl \times q_{sc}}{I} = \frac{\pi \times 0.001 \times 0.4 \times (1.52 \times 10^6)}{190}$$

or,
$$V_b = 10.05 \text{ V}$$

6.

Water is boiled at the rate of 25 kg/h in a polished copper pan, 280 mm in diameter, at atmospheric pressure. Assuming nucleate boiling conditions, calculate the temperature of the bottom surface of the pan.

Solution. Given: $m = 25 \text{ kg/h}$; $D = 280 \text{ mm} = 0.28 \text{ m}$

The properties of water at atmospheric pressure are:

$t_{sat} = 100^\circ\text{C}$; $\rho_l = 958.4 \text{ kg/m}^3$; $\rho_v = 0.5955 \text{ kg/m}^3$; $c_{pl} = 4220 \text{ J/kg K}$; $\mu_l = 279 \times 10^{-6}$; $Pr_l = 1.75$; $h_{fg} = 2257 \text{ kJ/kg}$; $\sigma = 58.9 \times 10^{-3} \text{ N/m}$; $n = 1$ (for water)

The temperature of the bottom surface, t_s :

Excess temperature $\Delta t_e = t_s - t_{sat}$

For nucleate boiling (assumed), the following correlation holds good:

$$q_s = \mu_l \cdot h_{fg} \left[\frac{g (\rho_l - \rho_v)}{\sigma} \right]^{0.5} \left[\frac{c_{pl} \cdot \Delta t_e}{C_{sl} \cdot h_{fg} \cdot Pr_l^n} \right]^3$$

For polished copper pan, $C_{sl} = 0.013$

$$\text{or, } \Delta t_e = \left[\frac{q_s}{\mu_l \cdot h_{fg}} \left\{ \frac{\sigma}{g (\rho_l - \rho_v)} \right\}^{0.5} \right]^{0.333} \left[\frac{C_{sl} \cdot h_{fg} \cdot Pr_l}{c_{pl}} \right]$$

$$\text{Here, } q_s = \text{Surface heat flux} = \frac{Q}{A} = \frac{m h_{fg}}{A}$$

(where, m = Rate of water evaporation)

$$\text{or, } q_s = \frac{25 \times (2257 \times 10^3)}{3600 \times \left(\frac{\pi}{4} \times 0.28^2 \right)} = 254544 \text{ W/m}^2$$

$$\therefore \Delta t_e = \left[\frac{254544}{279 \times 10^{-6} \times 2257 \times 10^3} \left\{ \frac{58.9 \times 10^{-3}}{9.81 (958.4 - 0.5955)} \right\}^{0.5} \right]^{0.333} \left[\frac{0.013 \times 2257 \times 10^3 \times 1.75}{4220} \right]$$

$$= [404.23 \times 0.0025]^{0.333} \times 12.16 = 12.2$$

i.e., $\Delta t_e = t_s - t_{sat} = 12.2$

or, $t_s = 12.2 + t_{sat} = 12.2 + 100 = \mathbf{112.2^\circ C (Ans.)}$

7.

Water at atmospheric pressure is to be boiled in polished copper pan. The diameter of the pan is 350 mm and is kept at 115°C. Calculate the following:

- (i) *Power of the burner;*
- (ii) *Rate of evaporation in kg/h;*
- (iii) *Critical heat flux for these conditions.*

Solution. Given : $D = 350 \text{ mm} = 0.35 \text{ m}$, $t_s = 115^\circ \text{C}$, $t_{sat} = 100^\circ \text{C}$

The thermo-physical properties of water (from table) at 100°C are:

$$\rho_l (= \rho_f) = 958.4 \text{ kg/m}^3; \rho_v = 0.5955 \text{ kg/m}^3; c_{pl} (= c_{pf}) = 4220 \text{ J/kg K};$$

$$\mu_l (= \mu_f) = 279 \times 10^{-6} \text{ Ns/m}^2$$

$$Pr_l (= Pr_f) = 1.75; h_{fg} = 2257 \text{ kJ/kg}; n = 1; \sigma = 58.9 \times 10^{-3} \text{ N/m}$$

The excess temperature, $\Delta t_e = t_s - t_{sat} = 115 - 100 = 15^\circ \text{C}$

(i) Power of the burner to maintain boiling:

As per boiling curve, for $\Delta t_e = 15^\circ\text{C}$, nucleate pool boiling will occur and for this the following correlation holds good:

$$q_s = \mu_l \cdot h_{fg} \left[\frac{g (\rho_l - \rho_v)}{\sigma} \right]^{0.5} \left[\frac{c_{pl} \cdot \Delta t_e}{C_{sl} \cdot h_{fg} \cdot Pr_l^n} \right]^3$$

For polished copper pan, $C_{sl} = 0.013$

Refer table 1

Substituting the values in the above eqn. we get

$$\begin{aligned} q_s &= 279 \times 10^{-6} \times (2257 \times 10^3) \left[\frac{9.81 (958.4 - 0.5955)}{58.9 \times 10^{-3}} \right]^{0.5} \left[\frac{4220 \times 15}{0.013 \times 2257 \times 10^3 \times 1.75} \right]^3 \\ &= 629.7 \times 399.4 \times 1.873 \\ &= 471.06 \times 10^3 \text{ W/m}^2 = 471.06 \text{ kW/m}^2 \end{aligned}$$

The boiling heat transfer rate (power of the burner) is given by

$$Q = 471.06 \times \frac{\pi}{4} \times (0.35)^2 = \mathbf{45.32 \text{ kW (Ans.)}}$$

(ii) Rate of evaporation, m_w :

Under steady state conditions, all the heat added to the pan will result in evaporation of water.

Thus,

$$\begin{aligned} Q &= m_w \times h_{fg} \\ \text{or, } m_w &= \frac{Q}{h_{fg}} = \frac{45.32 \times 10^3}{2257 \times 10^3} = 0.02 \text{ kg/s} = \mathbf{72 \text{ kg/h (Ans.)}} \end{aligned}$$

(iii) Critical heat flux, q_{sc} :

$$\begin{aligned} q_{sc} &= 0.18 (\rho_v)^{1/2} h_{fg} [g\sigma (\rho_l - \rho_v)]^{1/4} \\ &= 0.18 (0.5955)^{1/2} \times 2257 \times 10^3 [9.81 \times 58.9 \times 10^{-3} (958.4 - 0.5955)]^{1/4} \\ &= 1.52 \times 10^6 \text{ W/m}^2 = \mathbf{1.52 \text{ MW/m}^2 \text{ (Ans.)}} \end{aligned}$$

8. A metal-clad heating element of 10 mm diameter and of emissivity 0.92 is submerged in a water bath horizontally. If the surface temperature of the metal is 260°C under steady boiling conditions, calculate the power dissipation per unit length for the heater. Assume that water is exposed to atmospheric pressure and is at a uniform temperature.

Solution. Given : $D = 10 \text{ mm} = 0.01 \text{ m}$, $\epsilon = 0.92$, $t_s = 260^\circ\text{C}$

The thermo-physical properties of water at 100°C from table are :

$$\rho_l = \rho_f = 958.4 \text{ kg/m}^3; h_{fg} = 2257 \text{ kJ/kg}$$

The thermo-physical properties of vapour at 260°C from table are:

$$\rho_v = 4.807 \text{ kg/m}^3, c_{pv} = 2.56 \text{ kJ/kg K}, k = 0.0331 \text{ W/mK};$$

$$\mu_v = \mu_g = 14.85 \times 10^{-6} \text{ Ns/m}^2$$

Power dissipation per unit length for the heater:

The excess temperature $\Delta t_e = t_s - t_{sat} = 260 - 100 = 160^\circ\text{C}$

As per boiling curve, at $\Delta t_e = 160^\circ\text{C}$, there exists a film pool boiling condition. In this case, the heat transfer is due to both convection and radiation.

The heat transfer coefficient, h (approximate) is calculated from the equation:

$$h = h_{conv} + \frac{3}{4} h_{rad} \quad \dots[\text{Eqn. (9.13)}]$$

The convective heat transfer coefficient,

$$h_{conv} = 0.62 \left[\frac{k_v^3 \rho_v (\rho_l - \rho_v) g (h_{fg} + 0.4 c_{pv} \Delta t_e)}{\mu_v D \Delta t_e} \right]^{1/4} \quad \dots[\text{Eqn. (9.14)}]$$
$$= 0.62 \left[\frac{(0.0331)^3 \times 4.807 (958.4 - 4.807) \times 9.81 \times (2257 \times 10^3 + 0.4 \times 2.56 \times 10^3 \times 160)}{14.85 \times 10^{-6} \times 0.01 \times 160} \right]^{1/4}$$

or, $h_{conv} = 395.84 \text{ W/m}^2\text{C}$

The radiation heat transfer coefficient,

$$h_{rad} = \frac{5.67 \times 10^{-8} \epsilon (T_s^4 - T_{sat}^4)}{(T_s - T_{sat})} \quad \dots[\text{Eqn. (9.15)}]$$
$$= \frac{5.67 \times 10^{-8} \times 0.92 [(260 + 273)^4 - (100 + 273)^4]}{[(260 + 273) - (100 + 273)]}$$

or, $h_{rad} = 20 \text{ W/m}^2\text{C}$

$$\therefore h = 395.84 + \frac{3}{4} \times 20 = 410.4 \text{ W/m}^2\text{C}$$

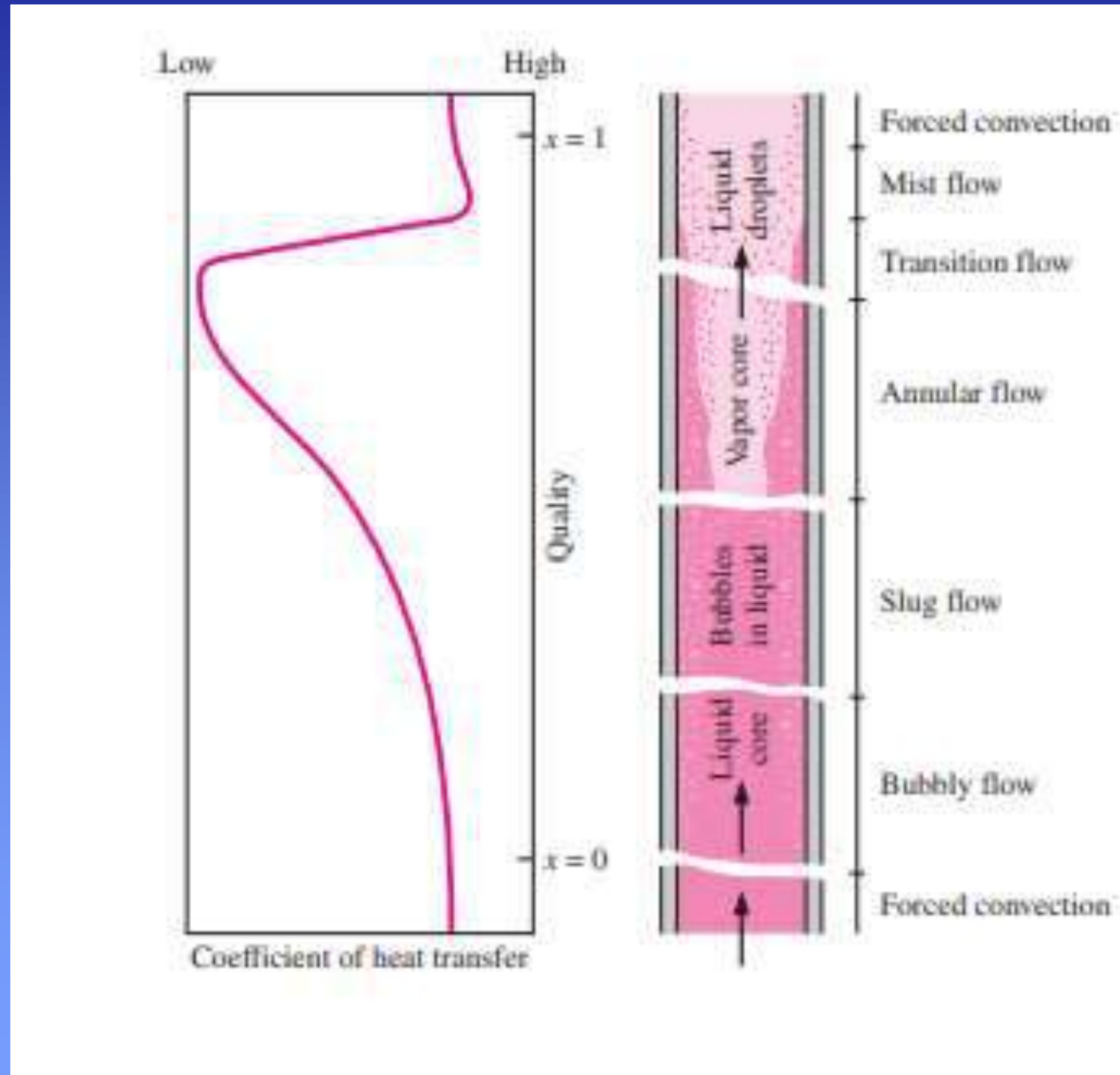
Hence the power dissipation per unit length for the heater

$$= h \times (\pi D \times 1) \times (260 - 100)$$
$$= 410.4 \times \pi \times 0.01 \times 160 = 2062.9 \text{ W/m or } \mathbf{2.063 \text{ kW/m (Ans.)}}$$

FLOW BOILING

- The fluid is forced to move by an external source such as a pump as it undergoes a phase-change process.
- Types: External or internal flow boiling
- **External flow boiling** over a plate or cylinder is similar to pool boiling, but the added motion increases both the nucleate boiling heat flux and the maximum heat flux considerably.
- **Internal flow boiling** commonly referred to as two-phase flow, is much more complicated in nature because there is no free surface for the vapor to escape, and thus both the liquid and the vapor are forced to flow together.

Different flow regimes encountered in flow boiling in a tube under forced convection



CONDENSATION HEAT TRANSFER

Condensation occurs when the temperature of a vapor is reduced below its saturation temperature T_{sat}

TYPES

- **Filmwise condensation:** The condensate wets the surface and forms a liquid film on the surface that slides down under the influence of gravity. The thickness of the liquid film increases in the flow direction as more vapor condenses on the film.
- **Dropwise condensation:** The condensed vapor forms droplets on the surface instead of a continuous film, and the surface is covered by countless droplets of varying diameters.

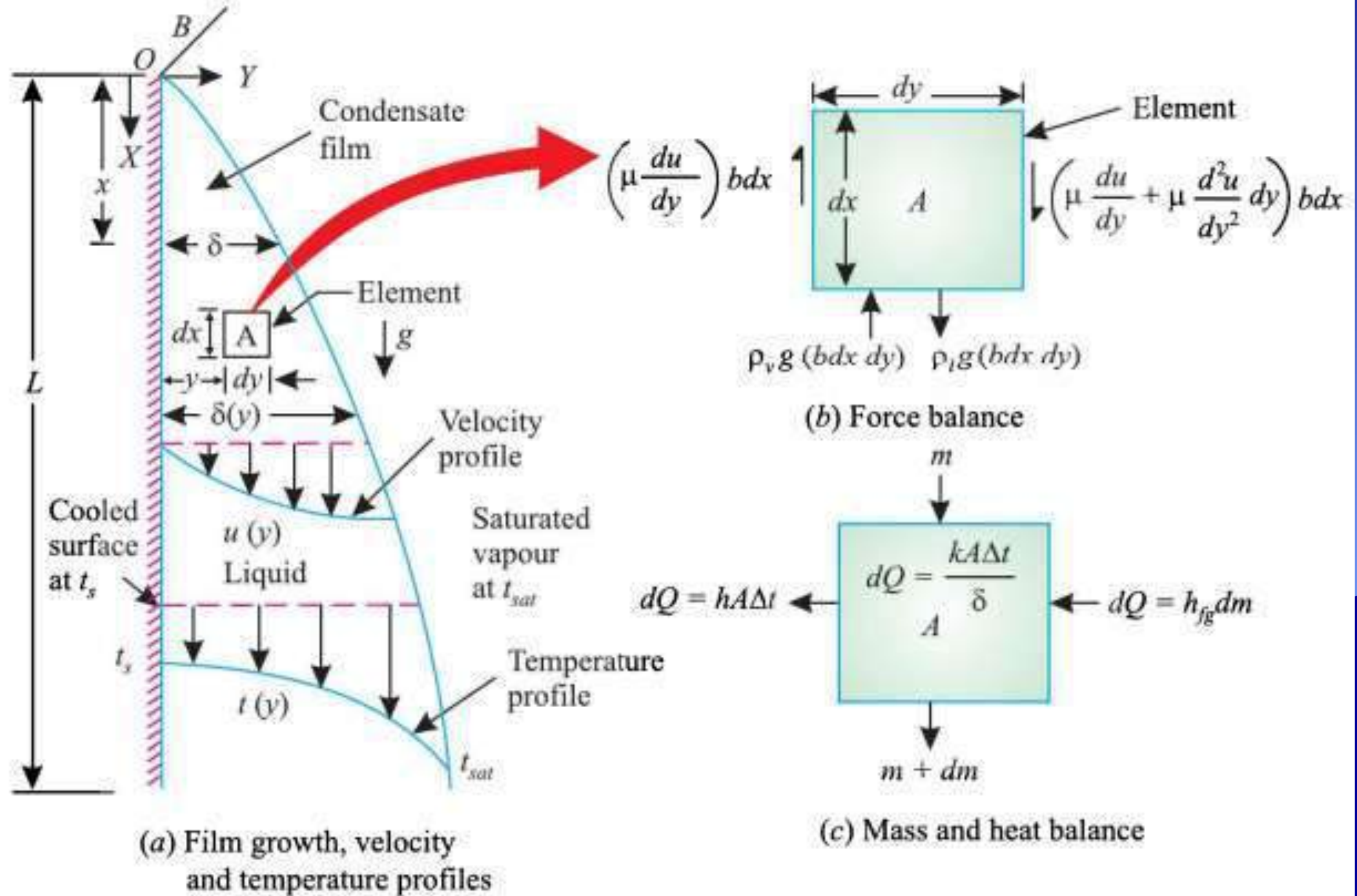
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NUSSELT'S THEORY OF CONDENSATION

Nusselt's analysis of film condensation makes the following simplifying **assumptions**.

1. The film of the liquid formed flows under the action of gravity.
2. The condensate flow is *laminar* and the fluid properties are constant.
3. The liquid film is in good thermal contact with the cooling surface and, therefore, temperature at the inside of the film is taken equal to the surface temperature t_s . Further, the temperature at the liquid-vapour interface is equal to the saturation temperature t_{sat} at the prevailing pressure.
4. Viscous shear and gravitational forces are assumed to act on the fluid; thus normal viscous force and inertia forces are neglected.
5. The shear stress at the liquid-vapour interface is negligible. This means there is no velocity gradient at the liquid-vapour interface [*i.e.*, $\left(\frac{\partial u}{\partial y} \right)_{y=\delta} = 0$].
6. The heat transfer across the condensate layer is by pure conduction and temperature distribution is linear.
7. The condensing vapour is entirely clean and free from gases, air and non-condensing impurities.

NUSSELT'S THEORY OF CONDENSATION



FILMWISE CONDENSATION

When the plate on which condensation occurs is *quite long* or when the *liquid film is vigorous enough*, the condensate flow may become turbulent. The *turbulent results in higher heat transfer rates because heat is now transferred not only by condensation but also by eddy diffusion*. The transition criterion may be expressed in terms of Reynolds number defined as,

$$Re = \frac{\rho_l u_m D_h}{\mu_l}$$

where, D_h = Hydraulic diameter

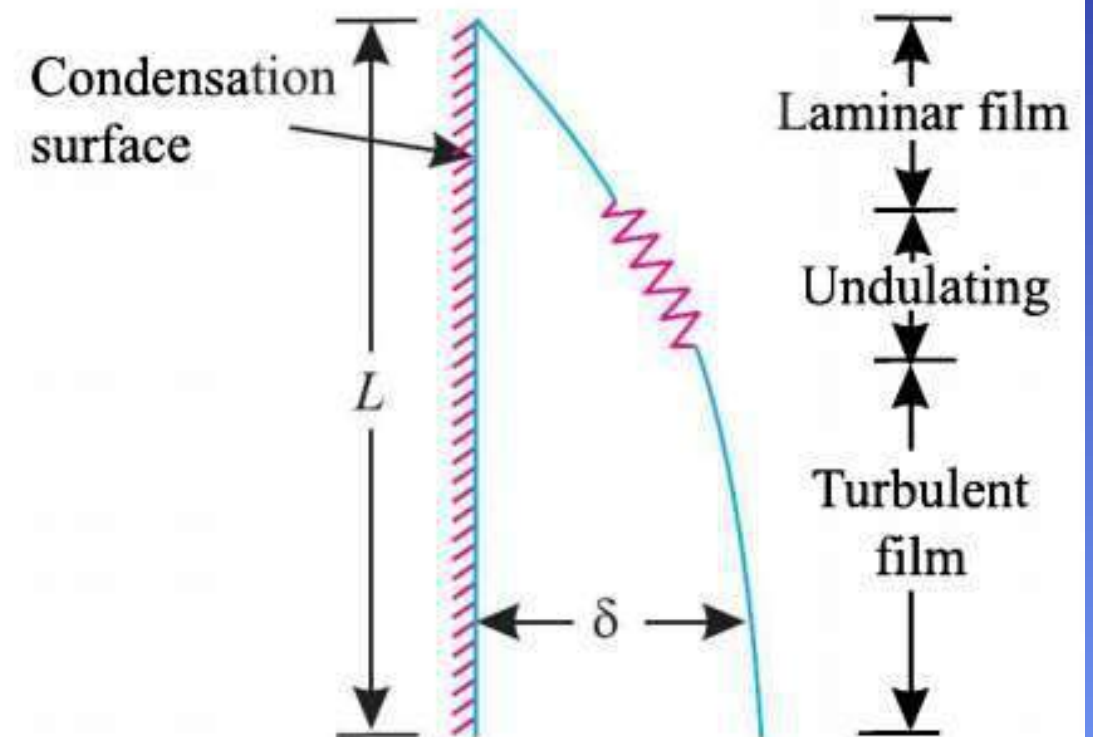


Fig. 9.10. Regions of film condensation on a vertical surface.

$$= 4 \times \frac{\text{cross-sectional area of fluid flow}}{\text{wetted perimeter}} = \frac{4A}{P}, \text{ and}$$

u_m = Mean or average velocity of flow.

$$Re = \frac{\rho_l u_m \times 4 A_c}{P \times \mu_l} = \frac{4 m}{P \mu_l} \quad \dots(9.35)$$

where, $m = \rho A u_m$

For a vertical plate of unit depth, $P = 1$, the Reynolds number is sometimes expressed in terms of the mass flow rate per unit depth of plate Γ , so that

$$Re = \frac{4 \Gamma}{\mu_l} \quad \dots(9.36)$$

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with $\Gamma = 0$, at the top of the plate and Γ increasing with x .

The Reynolds number may also be related to heat transfer coefficient as follows:

$$Q = \bar{h} A_s (t_{sat} - t_s) = \dot{m} h_{fg}$$

or,

$$\dot{m} = \frac{Q}{h_{fg}} = \frac{\bar{h} A_s (t_{sat} - t_s)}{h_{fg}}$$

or,

$$Re = \frac{4\bar{h} A_s (t_{sat} - t_s)}{h_{fg} P\mu_l} \quad \dots(9.37)$$

For the plate, $A = L \times B$ and $P = B$, where L and B are height and width of plate, respectively. Thus,

$$Re = \frac{4\bar{h} L (t_{sat} - t_s)}{h_{fg} \mu_l} \quad \dots(9.38)$$

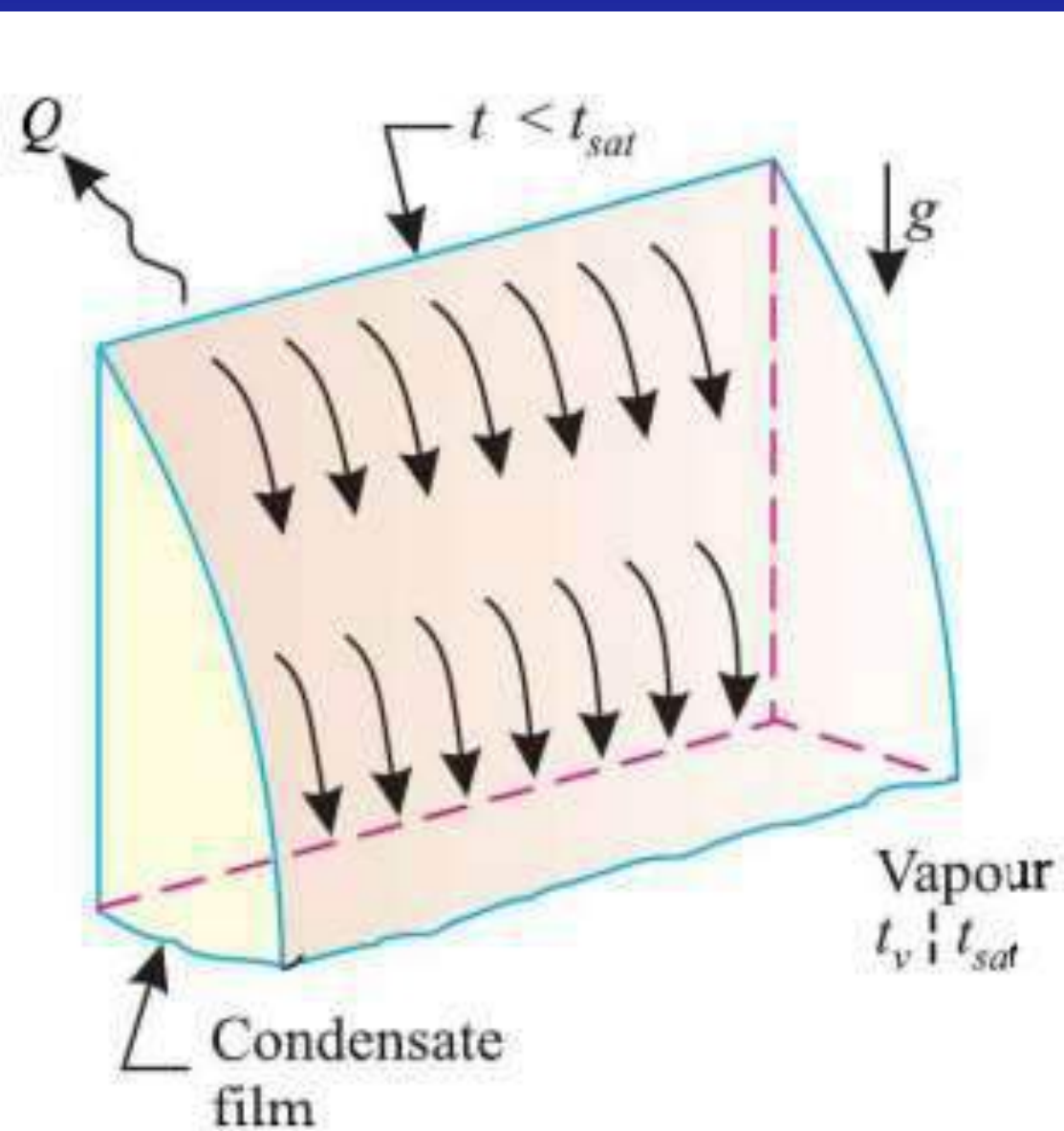
When the value of Re exceeds 1800 (approximately), the turbulence will appear in the liquid film. For $Re > 1800$, the following correlation is used:

$$\bar{h} (= h_{turb}) = 0.0077 \left[\frac{\rho_l (\rho_l - \rho_v) k^3 g}{\mu_l^2} \right]^{1/3} (R_l)^{0.4} \quad \dots(9.39)$$

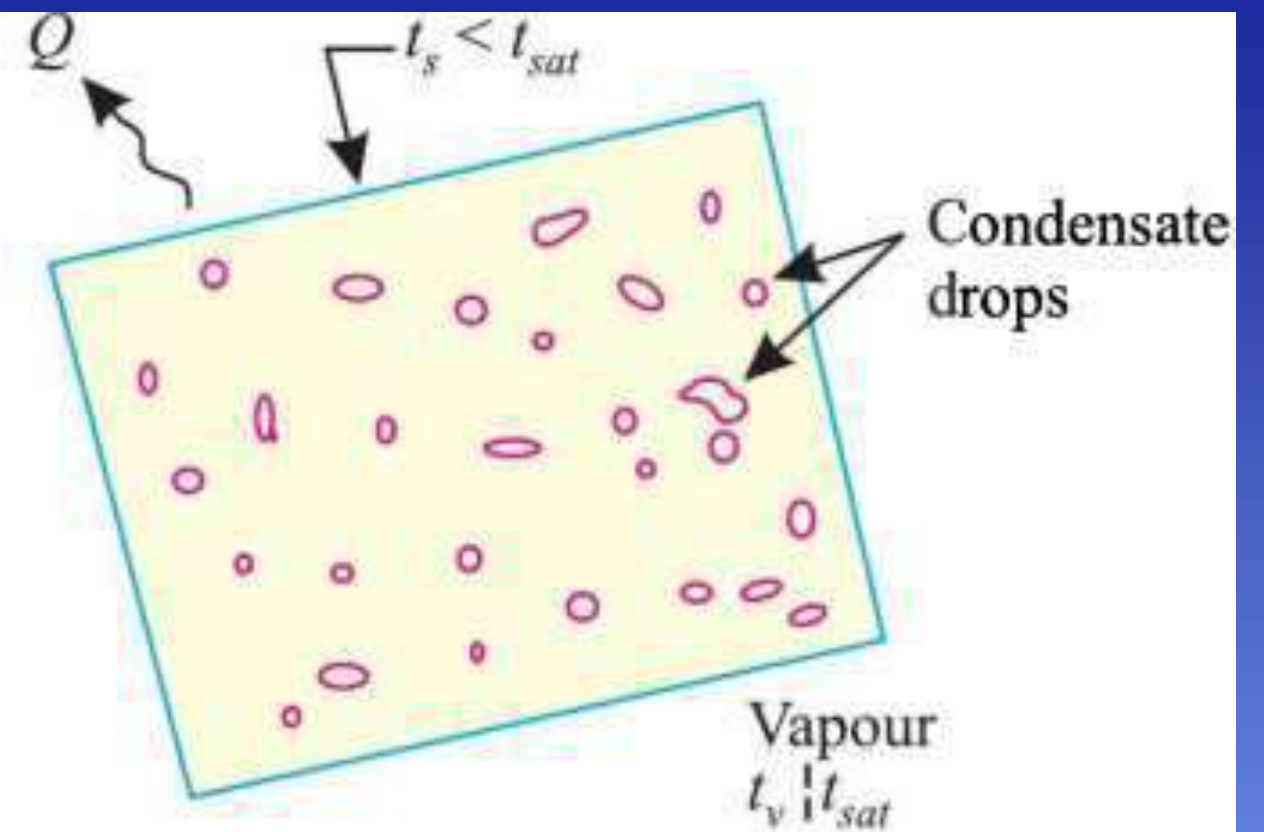
DROPSWISE CONDENSATION

In 'dropwise condensation' *the vapour condenses into small liquid droplets of various sizes which fall down the surface in random fashion.* The drops form in cracks and pits on the surface, grow in size, break away from the surface, knock off other droplets and eventually run off the surface, without forming a film under the influence of gravity. Fig. 9.6 (ii) shows the dropwise condensation on a vertical plate.

In this type of condensation, a large portion of the area of solid surface is directly exposed to vapour without an insulating film of condensate liquid, consequently *higher heat transfer rate* (to the order of 750 kW/m^2) *are achieved.* Dropwise condensation has been observed to occur either on highly polished surfaces, or on surfaces contaminated with impurities like fatty acids and organic compounds. This type of condensation gives coefficient of heat transfer generally *5 to 10 times larger than with film condensation.* Although dropwise condensation would be preferred to filmwise condensation yet *it is extremely difficult to achieve or maintain.* This is because most surfaces become 'wetted' after being exposed to condensing vapours over a period of time. Dropwise condensation can be obtained under controlled conditions with the help of certain additives to the condensate and various surface coatings but its commercial viability has not yet been approved. For this reason the *condensing equipment in use is designed on the basis of filmwise condensation.*



(i) Film condensation



(ii) Dropwise condensation

Fig. 9.6. Film and dropwise condensations on a vertical surface.

Example 9.8. Saturated steam at $t_{sat.} = 90^\circ\text{C}$ ($p = 70.14 \text{ kPa}$) condenses on the outer surface of a 1.5 m long 2.5 cm OD vertical tube maintained at a uniform temperature $T_\infty = 70^\circ\text{C}$. Assuming film condensation, calculate :

- (i) The local transfer coefficient at the bottom of the tube, and
- (ii) The average heat transfer coefficient over the entire length of the tube.

Properties of water at 80°C are : $\rho_l = 974 \text{ kg/m}^3$, $k_l = 0.668 \text{ W/m K}$, $\mu_l = 0.335 \times 10^{-3} \text{ kg/ms}$, $h_{fg} = 2309 \text{ kJ/kg}$, $\rho_v \ll \rho_l$

Solution. Given : $t_{sat} = 90^\circ\text{C}$ ($p = 70.14 \text{ kPa}$); $L = 1.5 \text{ m}$;
 $D = 2.5 \text{ cm} = 0.025 \text{ m}$; $t_s = 70^\circ\text{C}$.

Properties of water at 80°C $\left(t_f = \frac{90 + 70}{2} = 80^\circ\text{C} \right)$; $\rho_l = 974 \text{ kg/m}^3$;

$k = 0.668 \text{ W/m K}$; $\mu = 0.335 \times 10^{-3} \text{ kg/ms}$; $h_{fg} = 2309 \text{ kJ/kg}$ ($\rho_v \ll \rho_l$)

(i) The local heat transfer coefficient, h_x :

With usual notations, the local heat transfer coefficient for film condensation is given as :

$$h_x = \left[\frac{\rho_l (\rho_l - \rho_v) k^3 g h_{fg}}{4 \mu x (t_{sat} - t_s)} \right]^{1/4} \quad \dots[\text{Eqn. (9.26)}]$$

\therefore Local heat transfer coefficient at the bottom of the tube, $x = 1.5$ m, is

$$\begin{aligned} h_L (= h_{1.5}) &= \left[\frac{(974)^2 \times (0.668)^3 \times 9.81 (2309 \times 10^3)}{4 \times 0.335 \times 10^{-3} \times 1.5 (90 - 70)} \right]^{1/4} \quad (\text{as } \rho_v \ll \rho_l) \\ &= \left[\frac{6.4053 \times 10^{15}}{40.2} \right]^{1/4} = \mathbf{3552.9 \text{ W/m}^2 \text{ } ^\circ\text{C}} \quad (\text{Ans.}) \end{aligned}$$

(ii) Average heat transfer coefficient, \bar{h} :

$$\bar{h} = \frac{4}{3} h_L = \frac{4}{3} \times 3552.9 = \mathbf{4737.2 \text{ W/m}^2 \text{ } ^\circ\text{C}} \quad (\text{Ans.})$$

Example 9.9. Saturated steam at 120°C condenses on a 2 cm OD vertical tube which is 20 cm long. The tube wall is maintained at a temperature of 119°C . Calculate the average heat transfer coefficient and the thickness of the condensate film at the base of the tube. Assume Nusselt's solution is valid. Given :

$$p_{\text{sat}} = 1.985 \text{ bar}; \rho_w = 943 \text{ kg/m}^3; h_{fg} = 2202.2 \text{ kJ/kg};$$

$$k_w = 0.686 \text{ W/m K}; \mu = 237.3 \times 10^{-6} \text{ Ns/m}^2.$$

Solution. From Nusselt's solution, we have

$$\delta = \left[\frac{4k\mu (t_{\text{sat.}} - t_s) x}{\rho_l (\rho_l - \rho_v) g h_{fg}} \right]^{1/4} \quad \dots[\text{Eqn. (9.24)}]$$

or,
$$\delta_L = \left[\frac{4 \times 0.686 \times 237.3 \times 10^{-6} \times (120 - 119) \times 0.2}{(943)^2 \times 9.81 \times 2202.2 \times 10^3} \right]^{1/4},$$

neglecting ρ_v in comparison to ρ_l (or ρ_w)

$$= \left[\frac{0.0001302}{1.92 \times 10^{13}} \right]^{1/4} = 5.1 \times 10^{-5} \text{ m} \quad \text{or} \quad \mathbf{0.051 \text{ mm (Ans.)}}$$

Now,
$$h_L = \frac{k}{\delta_L} = \frac{0.686}{0.051 \times 10^{-3}} = 13451$$

\therefore Average heat transfer coefficient,

$$\bar{h} = \frac{4}{3} h_L = \frac{4}{3} \times 13451 = \mathbf{17934.67 \text{ W/m}^2\text{K}} \quad (\text{Ans.})$$

Example 9.10. A vertical cooling fin approximating a flat plate 40 cm in height is exposed to saturated steam at atmospheric pressure ($t_{\text{sat.}} = 100^\circ\text{C}$, $h_{\text{fg}} = 2257 \text{ kJ/kg}$). The fin is maintained at a temperature of 90°C . Estimate the following :

- (i) Thickness of the film at the bottom of the fin;
- (ii) Overall heat transfer coefficient; and
- (iii) Heat transfer rate after incorporating McAdam's correction.

The relevant fluid properties are :

$$\rho_l = 965.3 \text{ kg/m}^3$$

$$k_l = 0.68 \text{ W/m}^\circ\text{C}$$

$$\mu_l = 3.153 \times 10^{-4} \text{ N s/m}^2$$

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The following relations may be used :

$$\delta_x = \left[\frac{4k_l \mu_l (t_{sat.} - t_s)x}{gh_{fg} \rho_l (\rho_l - \rho_v)} \right]^{1/4}$$

$$\bar{h} = \frac{4}{3} \frac{k_l}{\delta_L}$$

Solution. Given : $L = 60 \text{ cm} = 0.6 \text{ m}$; $t_{sat.} = 100^\circ\text{C}$; $h_{fg} = 2257 \text{ kJ/kg}$;
 $t_s = 90^\circ\text{C}$; $\rho_l = 965.3 \text{ kg/m}^3$; $k_l = 0.68 \text{ W/m}^\circ\text{C}$;
 $\mu_l = 3.153 \times 10^{-4} \text{ N s/m}^2$

(i) Thickness of film at the bottom edge of the fin, δ_L :

$$\delta_x = \left[\frac{4k_l \mu_l (t_{sat.} - t_s)x}{gh_{fg} \rho_l (\rho_l - \rho_v)} \right]^{1/4} \quad \dots(\text{Given})$$

or,

$$\delta_L = \left[\frac{4k_l \mu_l (t_{sat.} - t_s)L}{gh_{fg} \rho_l^2} \right]^{1/4}, \text{ as } \rho_l \gg \rho_v$$

$$= \left[\frac{4 \times 0.68 \times 3.153 \times 10^{-4} (100 - 90) \times 0.4}{9.81 \times 2257 \times 10^3 \times (965.3)^2} \right]^{1/4} = \left[\frac{34.305 \times 10^{-4}}{2.063 \times 10^{13}} \right]^{1/4}$$

$$= 0.0001136 \text{ m} = \mathbf{0.1136 \text{ mm}} \quad (\text{Ans.})$$

(ii) Overall heat transfer coefficient, \bar{h} :

$$\bar{h} = \frac{4}{3} \frac{k_l}{\delta_L} = \frac{4}{3} \times \frac{0.68}{0.0001136} = \mathbf{7981.22 \text{ W/m}^2^\circ\text{C}} \quad (\text{Ans.})$$

(iii) Heat transfer rate with McAdam's correction :

With McAdam's correction, the value of \bar{h} is 20 percent higher. Hence heat transfer rate after incorporating McAdam's correction for unit width, is :

$$Q = 1.2 \times 7981.22 \times (0.4 \times 1) \times (100 - 90)$$

$$= 38309.8 \text{ W/m} \quad \text{or} \quad \mathbf{38.3098 \text{ kW per m width}} \quad (\text{Ans.})$$

Example 9.11. A vertical plate 500 mm high and maintained at 30°C is exposed to saturated steam at atmospheric pressure. Calculate the following:

- (i) The rate of heat transfer, and
- (ii) The condensate rate per hour per metre of the plate width for film condensation.

The properties of water film at the mean temperature are:

$$\rho = 980.3 \text{ kg/m}^3; k = 66.4 \times 10^{-2} \text{ W/m}^\circ\text{C}; \mu = 434 \times 10^{-6} \text{ kg/ms and } h_{fg} = 2257 \text{ kJ/kg.}$$

Assume vapour density is small compared to that of the condensate.

Solution. Given: $L = 500 \text{ mm} = 0.5 \text{ m}$; $B = 1 \text{ m}$; $t_s = 30^\circ\text{C}$.

(i) The rate of heat transfer per metre width, Q :

$$\bar{h} = 0.943 \left[\frac{\rho_l (\rho_l - \rho_v) k^3 g h_{fg}}{\mu L (t_{sat.} - t_s)} \right]^{1/4} \quad \dots[\text{Eqn. (9.29)}]$$

$$= 0.943 \left[\frac{\rho_l^2 k^3 g h_{fg}}{\mu L (t_{sat.} - t_s)} \right]^{1/4} \quad \dots\text{neglecting } \rho_v [(\rho_v \ll \rho_l \dots \text{given})]$$

or,

$$\bar{h} = 0.943 \left[\frac{(980.3)^2 \times (66.4 \times 10^{-2})^3 \times 9.81 \times (2257 \times 10^3)}{434 \times 10^{-6} \times 0.5 (100 - 30)} \right]^{1/4}$$

$$= 0.943 \left[\frac{6.229 \times 10^{12}}{0.0152} \right]^{1/4} = 4242.8 \text{ W/m}^2\text{°C}$$

$$\begin{aligned} \therefore Q &= \bar{h} A (t_{sat} - t_s) = h \times (L \times B) (t_{sat} - t_s) \\ &= 4242.8 \times (0.5 \times 1) (100 - 30) = 148498 \text{ W} \\ &= \frac{148498 \times 3600}{1000} = \mathbf{534.59 \times 10^3 \text{ kJ/h}} \end{aligned}$$

(ii) The condensate rate per meter width, m :

$$m = \frac{Q}{h_{fg}} = \frac{534.59 \times 10^3}{2257} = \mathbf{236.86 \text{ kg/h (Ans.)}}$$

Example 9.12. A vertical plate 350 mm high and 420 mm wide, at 40°C, is exposed to saturated steam at 1 atm. Calculate the following:

- (i) The film thickness at the bottom of the plate;
- (ii) The maximum velocity at the bottom of the plate;
- (iii) The total heat flux to the plate.

Assume vapour density is small compared to that of the condensate.

Solution. Given: $t_s = 40^\circ\text{C}$; $t_{sat} = 100^\circ\text{C}$, $L = 350 \text{ mm} = 0.35 \text{ m}$, $B = 420 \text{ mm} = 0.42 \text{ m}$.

The properties will be evaluated at the film temperature, i.e., the average of t_{sat} and t_s ;

$$t_f = \frac{100 + 40}{2} = 70^\circ\text{C}; \text{ further } h_{fg} \text{ is evaluated at } 100^\circ\text{C}.$$

The properties at 70°C are:

$\rho_l = 977.8 \text{ kg/m}^3$; $\mu = 0.4 \times 10^{-3} \text{ kg/ms}$; $k = 0.667 \text{ W/m}^\circ\text{C}$ and $h_{fg} = 2257 \text{ kJ/kg}$.

(i) The film thickness at the bottom of the plate, δ :

$$\delta = \left[\frac{4k\mu(t_{sat} - t_s)x}{g\rho_l(\rho_l - \rho_v)h_{fg}} \right]^{1/4} \quad \dots[\text{Eqn. (9.24)}]$$

$$= \left[\frac{4k\mu(t_{sat} - t_s)x}{g \cdot \rho_l^2 h_{fg}} \right]^{1/4} \quad \text{Neglecting } \rho_v, \rho_v \ll \rho_l \quad \dots(\text{given})$$

$$\text{or, } \delta = \left[\frac{4 \times 0.667 \times 0.4 \times 10^{-3} (100 - 40) \times 0.35}{9.81 \times (977.8)^2 \times 2257 \times 10^3} \right]^{1/4} = 1.8 \times 10^{-4} \text{ m} = \mathbf{0.18 \text{ mm}}$$

($\because x = l = 0.35 \text{ m}$ in this case)

(ii) The maximum velocity at the bottom of the plate, u_{max} :

$$u = \frac{(\rho_l - \rho_v) g}{\mu} \left(\delta y - \frac{y^2}{2} \right) \quad \dots[\text{Eqn. (9.17)}]$$

$$= \frac{\rho_l g}{\mu} \left(\delta y - \frac{y^2}{2} \right) \quad \dots\text{neglecting } \rho_v$$

At $y = \delta$, $u = u_{max}$., therefore,

$$u_{max} = \frac{\rho_l g \delta^2}{2\mu} = \frac{977.8 \times 9.81 \times (1.8 \times 10^{-4})^2}{2 \times 0.4 \times 10^{-3}} = \mathbf{0.338 \text{ m/s}} \quad \mathbf{(Ans.)}$$

(iii) The total heat flux to the plate, Q :

$$\bar{h} = 0.943 \left[\frac{\rho_l (\rho_l - \rho_v) k^3 g h_{fg}}{\mu L (t_{sat} - t_s)} \right]^{1/4} \quad \dots[\text{Eqn. (9.29)}]$$

$$= 0.943 \left[\frac{\rho_l^2 k^3 g h_{fg}}{\mu L (t_{sat} - t_s)} \right]^{1/4} \quad \dots \text{neglecting } \rho_v$$

or,

$$\bar{h} = 0.943 \left[\frac{(977.8)^2 \times (0.667)^3 \times 9.81 \times 2257 \times 10^3}{0.4 \times 10^{-3} \times 0.35 (100 - 40)} \right]^{1/4}$$

$$= 0.943 \left[\frac{6.282 \times 10^{12}}{8.4 \times 10^{-3}} \right]^{1/4} = 4931.35 \text{ W/m}^\circ\text{C}$$

The total heat flux is given by

$$\begin{aligned} Q &= \bar{h} A (t_{sat} - t_s) = \bar{h} \times (L \times B) (t_{sat} - t_s) \\ &= 4931.35 \times 0.35 \times 0.42 \times (100 - 40) \\ &= 43494 \text{ W or } \mathbf{43.494 \text{ kW}} \quad \mathbf{(Ans.)} \end{aligned}$$

Example 9.13. Vertical flat plate in the form of fin is 600 m in height and is exposed to steam at atmospheric pressure. If surface of the plate is maintained at 60°C, calculate the following:

- (i) The film thickness at the trailing edge of the film,
- (ii) The overall heat transfer coefficient,
- (iii) The heat transfer rate, and
- (iv) The condensate mass flow rate.

Assume laminar flow conditions and unit width of the plate.

Solution. Given : $L = 600 \text{ mm} = 0.6 \text{ m}$; $t_s = 100^\circ\text{C}$;

The properties of vapour at atmospheric pressure are:

$$t_{sat} = 100^\circ\text{C}, h_{fg} = 2257 \text{ kJ/kg}; \rho_v = 0.596 \text{ kg/m}^3.$$

The properties of saturated vapour at the mean film temperature $t_f = \frac{100 + 60}{2} = 80^\circ\text{C}$ are:

$$\rho_l = 971.8 \text{ kg/m}^3, k = 67.413 \times 10^{-2} \text{ W/m}^\circ\text{C}, \mu = 355.3 \times 10^{-6} \text{ Ns/m}^2 \text{ or kg/ms}$$

(i) The film thickness at the trailing edge of the plate, δ (at $x = L = 0.6 \text{ m}$):

$$\delta = \left[\frac{4 k \mu (t_{sat} - t_s) x}{\rho_l (\rho_l - \rho_v) g h_{fg}} \right]^{1/4} \quad \dots[\text{Eqn. (9.24)}]$$

$$\delta_L = \left[\frac{4 \times 67.413 \times 10^{-2} \times 355.3 \times 10^{-6} (100 - 60) \times 0.6}{971.8 (971.8 - 0.596) \times 9.81 \times (2257 \times 10^3)} \right]^{1/4}$$

$$\text{or, } \delta_L = \frac{0.02299}{2.08972 \times 10^{13}} = 1.82 \times 10^{-4} \text{ m} = \mathbf{0.182 \text{ mm}} \quad (\text{Ans.})$$

(ii) The overall heat transfer coefficient, \bar{h} :

$$\bar{h} = \frac{4}{3} h_L = \frac{4}{3} \frac{k}{\delta_L} = \frac{4}{3} \times \frac{67.413 \times 10^{-2}}{1.82 \times 10^{-4}} = 4938.68 \text{ W/m}^2\text{°C}$$

Using McAdam's correction which is 20% higher than Nusselt's result, we have

$$\begin{aligned}\bar{h} &= 4938.68 \times 1.2 \\ &= \mathbf{5926.4 \text{ W/m}^2\text{°C}} \quad \text{(Ans.)}\end{aligned}$$

(iii) The heat transfer rate, Q :

$$\begin{aligned}Q &= \bar{h} A_s (t_{sat} - t_s) = h \times (L \times B) (t_{sat} - t_s) \\ &= 5926.4 \times (0.6 \times 1) (100 - 60) = \mathbf{142233.6 \text{ W}}\end{aligned}$$

(iv) The condensate mass flow rate, m :

$$\begin{aligned} m &= \frac{Q}{h_{fg}} && \dots[\text{Eqn. (9.32)}] \\ &= \frac{142233.6}{2257 \times 10^3} = 0.063 \text{ kg/s or } \mathbf{226.8 \text{ kg/h}} && \text{(Ans.)} \end{aligned}$$

Let us check whether the flow is laminar or not.

$$\begin{aligned} Re &= \frac{4m}{\mu B} && \dots[\text{Eqn. (9.35)}] \\ &= \frac{4 \times 0.063}{355.3 \times 10^{-6} \times 1} = 709.26 < 1800 \end{aligned}$$

This shows that the assumption of laminar flow is correct.

Example 9.14. A vertical tube of 60 mm outside diameter and 1.2 m long is exposed to steam at atmospheric pressure. The outer surface of the tube is maintained at a temperature of 50°C by circulating cold water through the tube. Calculate the following:

- (i) The rate of heat transfer to the coolant, and
- (ii) The rate of condensation of steam.

Solution. Given : $D = 60 \text{ mm} = 0.06 \text{ m}$, $L = 1.2 \text{ m}$, $t_s = 50^\circ\text{C}$

Assuming the condensation film is laminar and noncondensable gases in steam are absent;

The mean film temperature $t_f = \frac{100 + 50}{2} = 75^\circ\text{C}$

The thermo-physical properties of water at 75°C are:

$$\rho_l = 975 \text{ kg/m}^3, \mu_l = 375 \times 10^{-6} \text{ Ns/m}^2, k = 0.67 \text{ W/m}^\circ\text{C}.$$

The properties of saturated vapour at $t_{sat} = 100^\circ\text{C}$ are :

$$\rho_v = 0.596 \text{ kg/m}^3, h_{fg} = 2257 \text{ kJ/kg}.$$

(i) **The rate of heat transfer, Q :**

For laminar condensation on a vertical surface

$$\bar{h} = 1.13 \left[\frac{\rho_l (\rho_l - \rho_v) k^3 g h_{fg}}{\mu L (t_{sat} - t_s)} \right]^{1/4} \quad \dots[\text{Eqn. (9.30)}]$$

or,
$$\bar{h} = 1.13 \left[\frac{975 (975 - 0.596) \times (0.67)^3 \times 9.81 \times (2257 \times 10^3)}{375 \times 10^{-6} \times 1.2 \times (100 - 50)} \right]^{1/4}$$

$$= 4627.3 \text{ W/m}^2\text{°C}$$

$$\begin{aligned} Q &= \bar{h} A_s (t_{sat} - t_s) = \bar{h} (\pi DL) (t_{sat} - t_s) \\ &= 4627.3 \times (\pi \times 0.06 \times 1.2) (100 - 50) = 52333.5 \\ &= \mathbf{52.333 \text{ kW (Ans.)}} \end{aligned}$$

(ii) **The rate of condensation of steam, m :**

The condensation rate is given by

$$m = \frac{Q}{h_{fg}} = \frac{52333.5}{2257 \times 10^3} = 0.0232 \text{ kg/s} = \mathbf{83.52 \text{ kg/h (Ans.)}}$$

Let us check the assumption of laminar film condensation by calculating Re .

$$Re = \frac{4m}{P \mu_l} \quad \dots[\text{Eqn. (9.35)}]$$

or,
$$Re = \frac{4 \times 0.0232}{\pi D \times 375 \times 10^{-6}} = \frac{4 \times 0.0232}{\pi \times 0.06 \times 375 \times 10^{-6}} = 1312.85$$

Since, $Re (= 1312.85) < 1800$, hence the flow is *laminar*.

Example 9.15. A horizontal tube of outer diameter 20 mm is exposed to dry steam at 100°C. The tube surface temperature is maintained at 84°C by circulating water through it. Calculate the rate of formation of condensate per metre length of the tube.

Solution. Given: $D = 20 \text{ mm} = 0.02 \text{ m}$, $t_s = 84^\circ\text{C}$; $t_{sat} = 100^\circ\text{C}$

The mean film temperature $t_f = \frac{100 + 84}{2} = 92^\circ\text{C}$

The properties of saturated liquid at 92°C are:

$$\rho_l = 963.4 \text{ kg/m}^3, \mu_l = 306 \times 10^{-6} \text{ Ns/m}^2; k = 0.677 \text{ W/m}^\circ\text{C}$$

The properties of saturated vapour at $t_{sat} = 100^\circ\text{C}$ are:

$$\rho_v = 0.596 \text{ kg/m}^3, h_{fg} = 2257 \text{ kJ/kg}$$

Rate of formation of condensate per metre length of the tube, m :

The average heat transfer coefficient is given by

$$\bar{h} = 0.725 \left[\frac{\rho_l (\rho_l - \rho_v) k^3 g h_{fg}}{\mu_l (t_{sat} - t_s) D} \right]^{1/4} \quad \dots[\text{Eqn. (9.40)}]$$

or,

$$\bar{h} = 0.725 \left[\frac{(963.4) (963.4 - 0.596) \times (0.677)^3 \times 9.81 \times (2257 \times 10^3)}{306 \times 10^{-6} (100 - 84) \times 0.02} \right]^{1/4}$$

$$= 11579.7 \text{ W/m}^2\text{°C}$$

The heat transfer per unit length is

$$\frac{Q}{L} = \bar{h} \times \pi D \times (t_{sat} - t_s)$$

$$= 11579.7 \times \pi \times 0.02 \times (100 - 84) = 11641.2 \text{ W}$$

Rate of formation of condensate per metre length of the tube,

$$\frac{m}{L} = \frac{Q/L}{h_{fg}} = \frac{11641.2}{2257 \times 10^3} = 5.157 \times 10^{-3} \text{ kg/s} = \mathbf{18.56 \text{ kg/h}} \quad \text{(Ans.)}$$