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Crisp and Fuzzy Sets: Basic Concepts and Set Operations

In the study of **fuzzy logic**, **sets** are a foundational concept. Sets are used to group or categorize elements based on certain properties. There are two main types of sets that are crucial in this context: **crisp sets** and **fuzzy sets**. Understanding these two types of sets and the operations that can be performed on them is fundamental for working with fuzzy logic and fuzzy systems.

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1. Crisp Sets

Definition of Crisp Sets:

A **crisp set** (also called a classical or traditional set) is a set in which an element either **belongs** or **does not belong** to the set. There is no ambiguity or partial membership in crisp sets. The elements of a crisp set are clearly defined.

- **Membership function**: In crisp sets, each element has a binary membership function: either **0** (not a member) or **1** (a member).
  - Example: Consider the set of "even numbers" EE = {2, 4, 6, 8, 10, …}. A number 4 belongs to the set of even numbers, and its membership value is 1. A number 3 does not belong, and its membership value is 0.
- **Representation**: A={x ∣ x∈U, x is an even number}A = \{ x \, | \, x \in U, \, x \text{ is an even number} \}, where UU is the universal set of all integers.

Properties of Crisp Sets:

- **Binary Membership**: An element is either fully in or fully out of the set.
- **Exclusivity**: There is no overlap or partial membership between elements of a set.

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2. Fuzzy Sets

Definition of Fuzzy Sets:

A **fuzzy set** is a set in which each element has a **degree of membership** that ranges from **0 to 1**. This allows for **partial membership**, meaning an element can belong to the set to some degree, but not necessarily fully.

- **Membership function**: In fuzzy sets, a membership function μA(x)\mu_A(x) assigns to each element xx a value between **0** and **1**. This value indicates the degree to which the element belongs to the fuzzy set.
  - Example: Consider the fuzzy set **"Tall people"**. The membership function might assign a person with a height of 170 cm a membership value of 0.7 (70% tall), and a person with a height of 190 cm a membership value of 1 (100% tall). A person with a height of 140 cm might have a membership value of 0.3.
- **Representation**: A={(x,μA(x)) ∣ x∈U, μA(x)∈[0,1]}A = \{ (x, \mu_A(x)) \, | \, x \in U, \, \mu_A(x) \in [0, 1] \}, where UU is the universal set of all people, and μA(x)\mu_A(x) indicates the degree of membership of person xx in the fuzzy set "Tall people".

Properties of Fuzzy Sets:

- **Partial Membership**: Elements can have degrees of membership between 0 and 1, meaning they can belong to the set to varying extents.
- **Overlapping Membership**: Elements can belong to multiple fuzzy sets at the same time, and the degrees of membership can be combined.

The set operations are performed on two or more sets to obtain a combination of elements as per the operation performed on them. In a set theory, there are three major types of operations performed on sets, such as:

1. Union of sets (∪)
2. Intersection of sets (∩)
3. Difference of sets (–)

Let us discuss these operations one by one.

Union of Sets

If two sets A and B are given, then the union of A and B is equal to the set that contains all the elements present in set A and set B. This operation can be represented as;

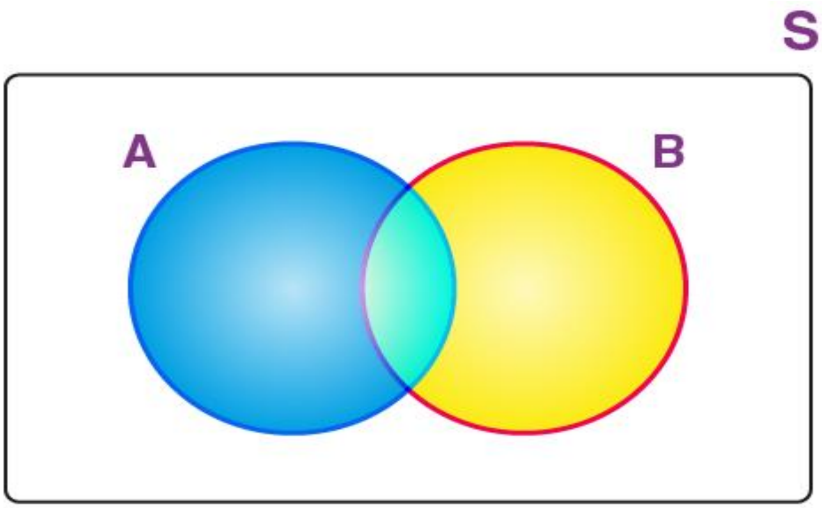
A ∪ B = {x: x ∈ A or x ∈ B}

Where x is the elements present in both sets A and B.

Example: If set A = {1,2,3,4} and B {6,7}

Then, Union of sets, A ∪ B = {1,2,3,4,6,7}

Venn Diagram of Union of sets



Intersection of Sets

If two sets A and B are given, then the intersection of A and B is the subset of universal set U, which consist of elements common to both A and B. It is denoted by the symbol '∩'. This operation is represented by:

A∩B = {x : x ∈ A and x ∈ B}

Where x is the common element of both sets A and B.

The intersection of sets A and B, can also be interpreted as:

A∩B = n(A) + n(B) – n(A∪B)

Where,

n(A) = cardinal number of set A,

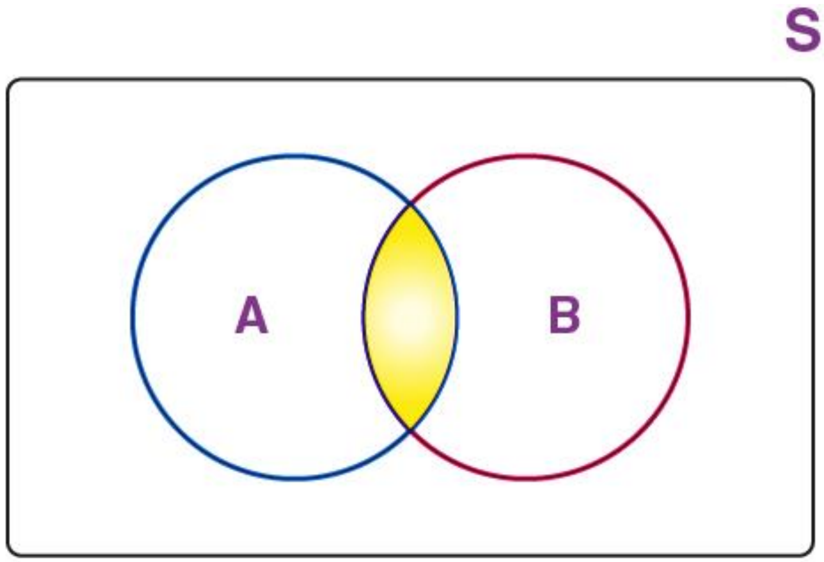
n(B) = cardinal number of set B,

n(A∪B) = cardinal number of union of set A and B.

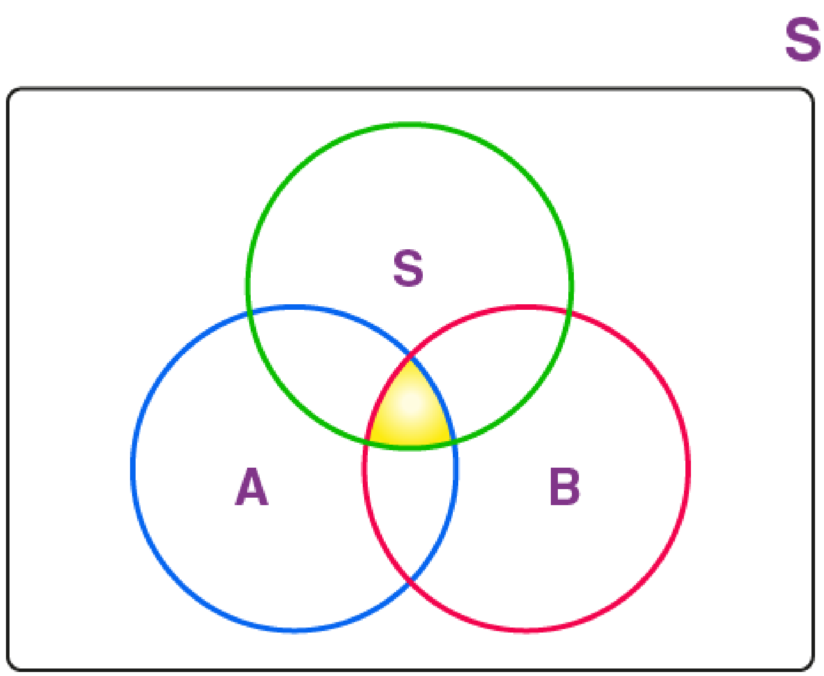
Example: Let A = {1,2,3} and B = {3,4,5}

Then, A∩B = {3}; because 3 is common to both the sets.

Venn Diagram of Intersection of sets



Intersection of Two Sets



Intersection of three sets

Difference of Sets

If there are two sets A and B, then the difference of two sets A and B is equal to the set which consists of elements present in A but not in B. It is represented by A-B.

Example: If A = {1,2,3,4,5,6,7} and B = {6,7} are two sets.

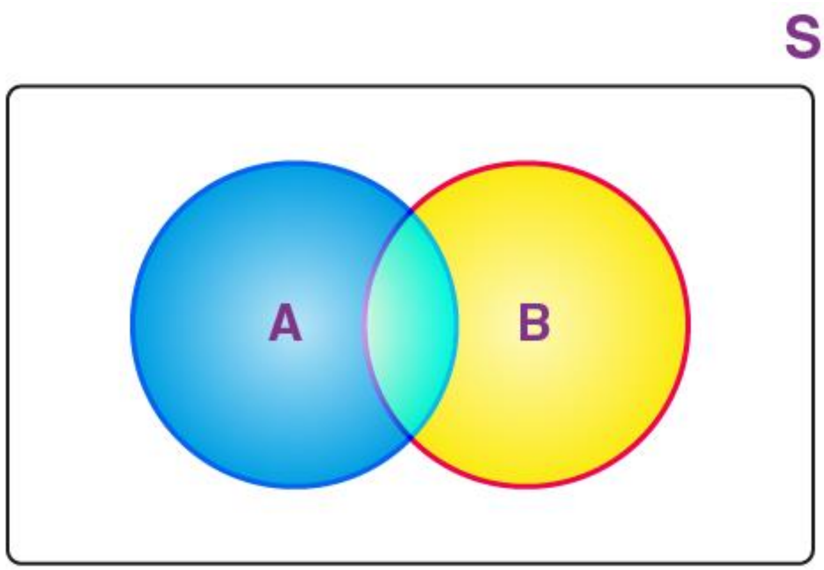
Then, the difference of set A and set B is given by;

A – B = {1,2,3,4,5}

We can also say, that the difference of set A and set B is equal to the intersection of set A with the complement of set B. Hence,

A−B=A∩B'

Venn Diagram of Difference of sets

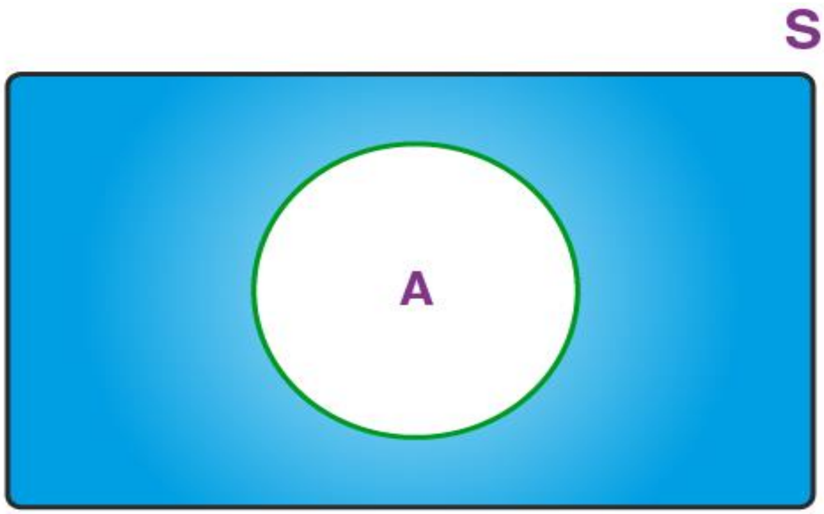


Complement of Set

If U is a universal set and X is any subset of U then the complement of X is the set of all elements of the set U apart from the elements of X.

X′ = {a : a ∈ U and a ∉ A}

Venn Diagram:



Example: U = {1,2,3,4,5,6,7,8}

A = {1,2,5,6}

Then, complement of A will be;

A’ = {3,4,7,8}