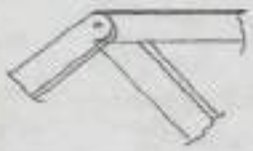
Subject Code & Name**: 23AST205-Aerospace Structures**

TOPIC: ***Plane Truss Analysis***

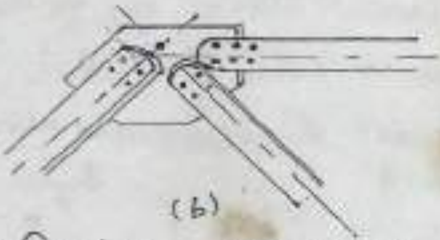
STATICALLY DETERMINATE STRUCTURES

ANALYSIS OF PLANE TRUSSES

A Truss, also called a pin-Jointed Frame, is an Idealised Skeletal (or) "Stick-like" Structure Composed of Slender rods joined together by Smooth pins at the Joints, also called "nodes". The joint of the Truss may be pinned or more likely welded (or) riveted together as shown.



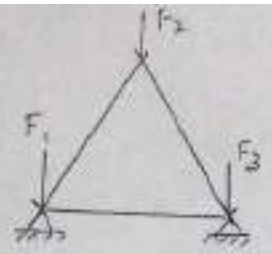
(a) Plane Frame held Together by a Single pin.



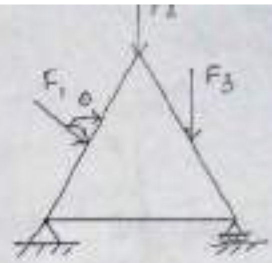
(b) Rivered (or) Bolted joint.

(c) Idealised joint.

The members of the Truss are Straight members and loads are applied only at the Joint.



Truss



Frame

Mathematical Condition for rigid or perfect Truss

A Truss Consist of a number of members which are Connected together and a form a Certain number of joints. For a truss to be rigid or perfect, the relationship between its number of members and the number of joints is given by

$$\boxed{m+3 = 2j} \quad -(1)$$

m – Number of members in the truss.

j – Number of joints.

Equation (1) Can be modified to

$$m = 2j - r$$

r – Number of external reactions.

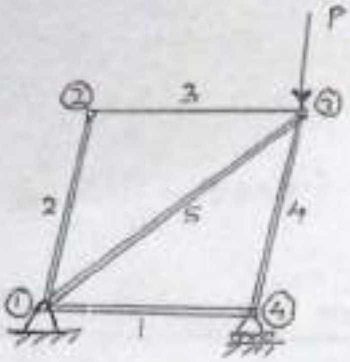
For a Simple Truss, the following equations is generally used to find the Statical Indeterminacy or the Statical Determinacy of the System.

$m = 2j - 3$; Statically determinate and Stable System.

$m > 2j - 3$; Statically Indeterminate and Stable System.

$m < 2j - 3$; Unstable System.

For example



$m = 5$

$j = 4$

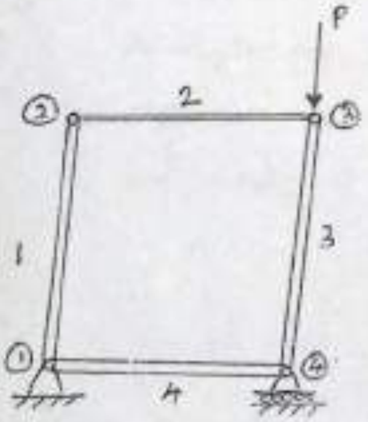
$\boxed{2j = m+3}$

$2(4) = 5+3$

$8 = 8$

Statical Determinate & Stable System.

Non Collapsible.



$m = 4.$

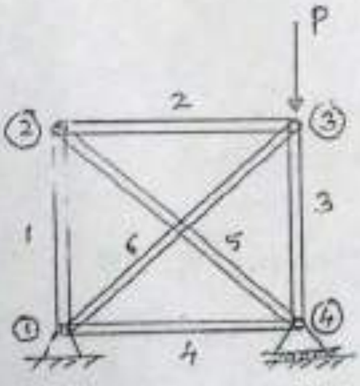
$j = 4.$

$2(4) = 4+3$

$8 > 7$

Unstable System.

Collapsible.



$m = 6$

$j = 4$

$2(4) = 6+3$

$8 < 7$

Statically Indeterminate & Stable

Over Rigid

8

In analysing the truss, it is important to remember the following principle theorem's

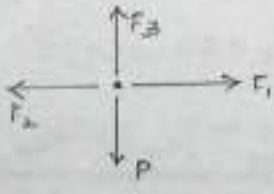
Theorem-I [For plane trusses (or) 2D]

At an unloaded truss joint with ~~three~~ two members, if any two members are collinear, the force in the third member is zero and the forces in the collinear member will be equal.

$F_3 = 0$.

Theorem 2 [For plane trusses]

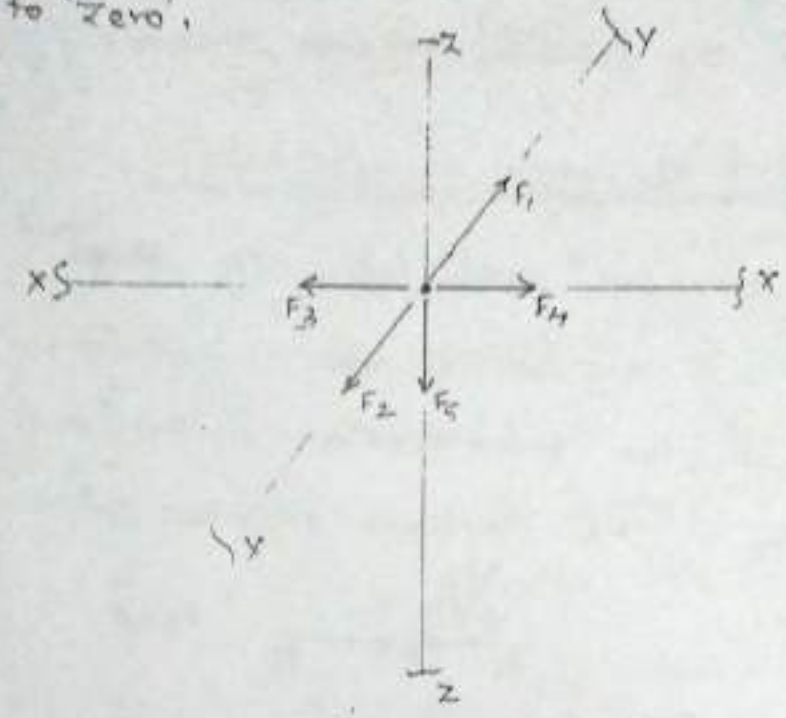
At a truss joint with three members, if two members are collinear and a load is applied in the direction of the third member, the force in the third member will be of the same magnitude as the applied load.



$F_3 = P$

THEOREM-3 [Space truss (or) 3D truss]

At a space truss joint, if the applied loads and all but one member lie in one plane, the force in the member which is not in that plane will be equal to 'Zero'.



The member of Force F_1, F_2, F_3 and F_4 lie in same plane xy, But F_5 lies in plane ⊥re to the plane xy in the direction of 'z'.

∴ Force in that member due to Forces in xy plane will be equal to 'Zero'.

$F_5 = 0$.

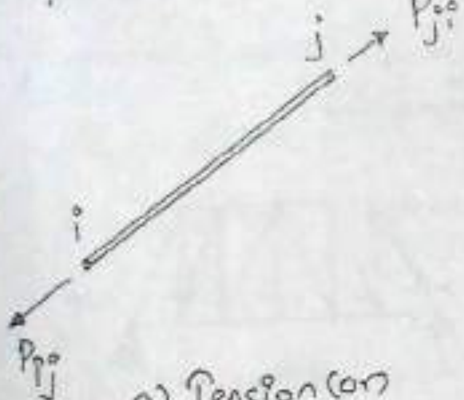
BASIC ASSUMPTIONS MADE FOR TRUSSES

1. The joints of a simple truss are assumed to be pin joined and frictionless.
2. Truss member are connected only at their ends.

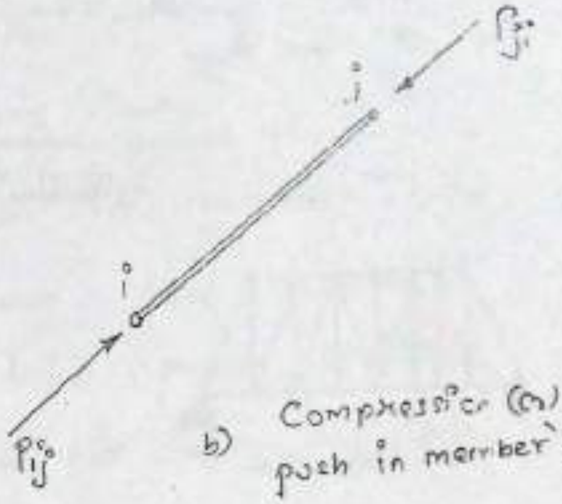
3. Trusses are loaded only at the joints.
4. The weight of the temperature may be neglected.

FREE BODY DIAGRAMS OF JOINTS & MEMBERS

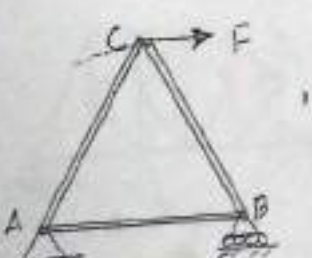
Based on the assumption, each truss member is subjected to the force only at the ends. Hence, members of trusses are two force members. In other words, the truss is created by joining such two force members by frictionless number of pins.



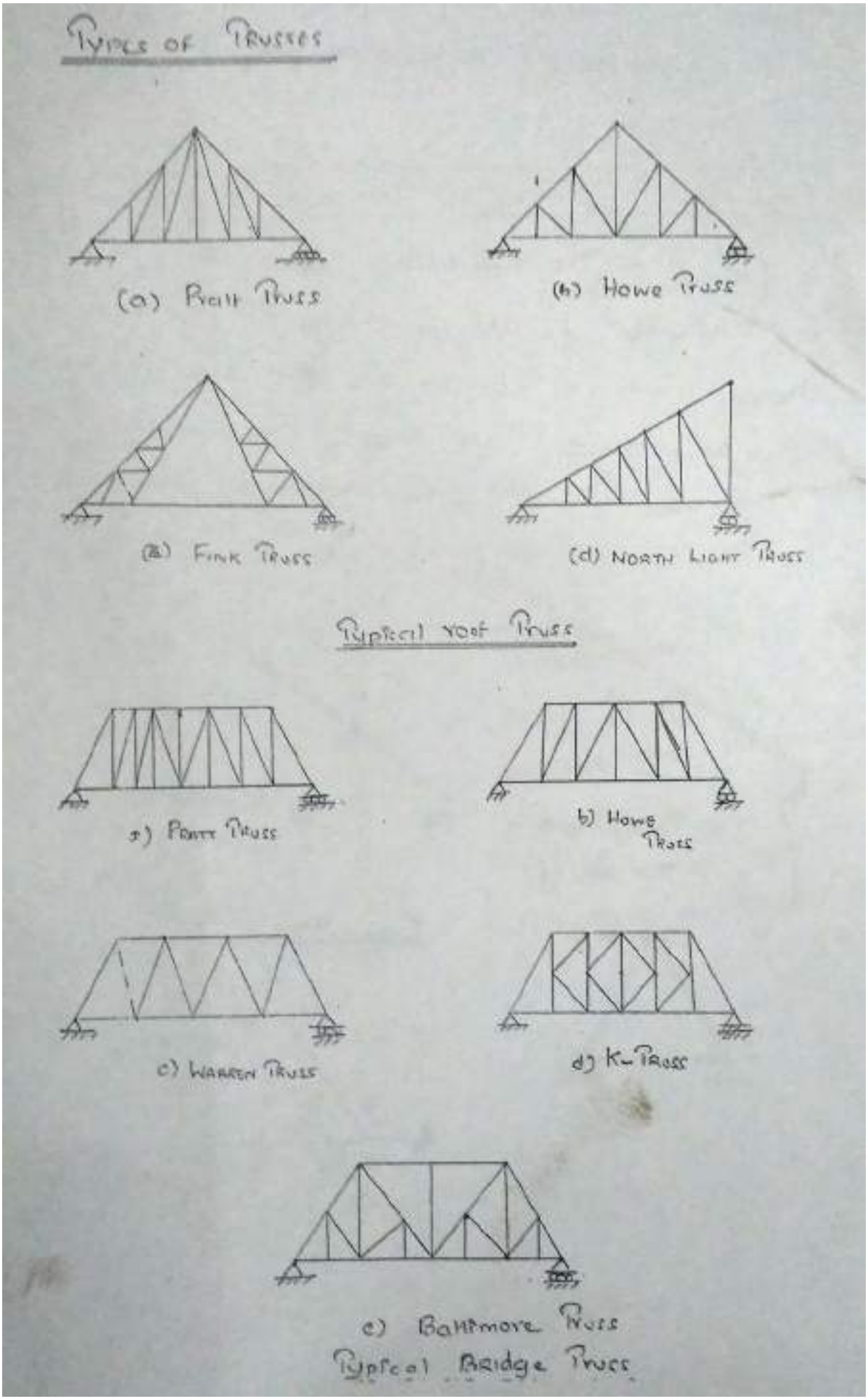
a) Tension (or) pull in 'ij'



b) Compression (or) push in member 'ij'



Free Body Diagram


Types of Trusses
(a) Pratt Truss
(b) Howe Truss
(c) Fink Truss
(d) North Light Truss
Typical roof Truss
a) Pratt Truss
b) Howe Truss
c) Warren Truss
d) K-Truss
e) Baltimore Truss
Typical Bridge Truss

Subject Code & Name: 23AST205-Aerospace Structures

TOPIC: ***METHOD OF JOINTS***

METHOD OF JOINTS

In this Method the Support reactions are worked out first. Then taking each joint, the force acting on it will be Considered as external forces, including the reactions (if any) and the forces in various members.

At every joint, two equations of equilibrium are available (ie)

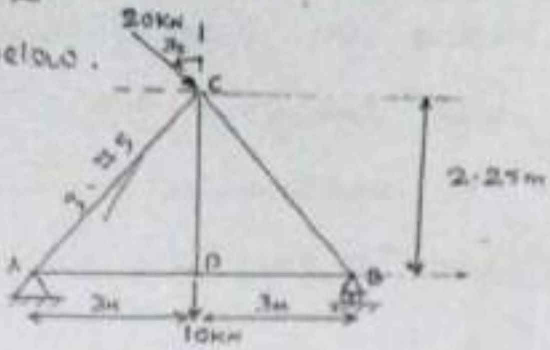
(i) $\Sigma H = 0$
(ii) $\Sigma V = 0$

At at one joint only two unknowns can be evaluated.

∴ The joint should be selected in such a way that only two unknown may be evaluated.

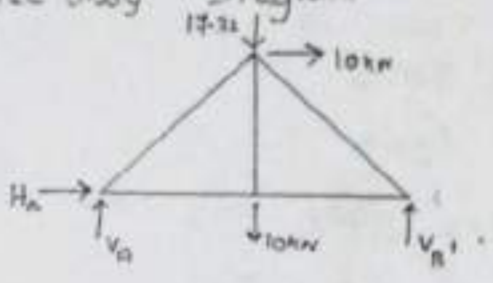
Repeating this process yields for Entire truss structure.

(1) Find the forces in all the members of the truss shown below.



Solution

Free body Diagram



To find Reactions (H_A, V_A) at A and (V_B) at B.

Using equations of equilibrium

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_x = 0$.

$\Sigma F_y = 0 \Rightarrow \quad V_A + V_B = 17.320 + 10$

$\boxed{V_A + V_B = 27.320}$ — ①

$\Sigma F_x = 0 \Rightarrow \quad H_A + 10 = 0$

$\boxed{H_A = -10\,kN}$ — ②

$\Sigma M_x = 0 \Rightarrow \quad M_A = 0$

$\Rightarrow (10 \times 3) + (17.32 \times 3) + (10 \times 2.25) = V_B \times 6$

$\boxed{V_B = 17.411\,kN}$ — (3)

$V_A + V_B = 27.320$

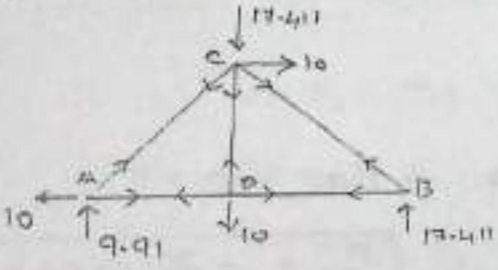
$V_A = 27.320 - 17.411$

$\boxed{V_A = 9.909 \text{ kN}}$

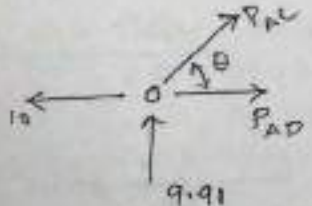
$V_A = 9.909$ KN

$V_B = 17.411$ KN

$H_A = -10$



Joint (A)



To find 'θ'

$\tan\theta = \frac{CD}{AD}$

$\boxed{\theta = 36.869}$

$AC^2 = AD^2 + DC^2$

$= (3)^2 + (2.25)^2$

$\boxed{AC = 3.75 m}$

$\Sigma F_x = 0$

$\Rightarrow -10 + P_{AD} + P_{AC} \cos(36.869) = 0$. —(1)

$\Sigma F_y = 0$

$9.91 + P_{AC} \sin(36.869) = 0.$

$P_{AC} (0.5999) = -9.91$

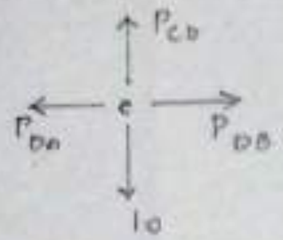
$\boxed{P_{AC} = -16.5170 \text{ kN}}$

$\Rightarrow$ Sub $P_{AC} = -16.517$ in (1)

$-10 + P_{AD} - (16.517)(0.8) = 0.$

$$\boxed{P_{AD} = 23.214\ kN}$$

At joint D



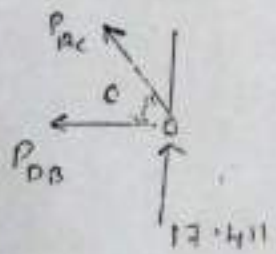
$\Sigma F_x = 0$

$-P_{DA} + P_{DB} = 0.$

$$\boxed{P_{CD} = 10\ kN}$$

$$\boxed{P_{DB} = +23.214\ kN}$$

At joint B



$17.411 + P_{BC} \sin\theta = 0$

$$\boxed{P_{BC} = -29.0189\ kN}$$

$P_{DB} + P_{BC} \cos\theta = 0.$

$-23.214 - P_{BC} \cos\theta = 0.$

$$-P_{BC} = \frac{-23.214}{0.8}$$

$$\boxed{P_{BC} = -29.0175\ kN}$$

Forces in Members

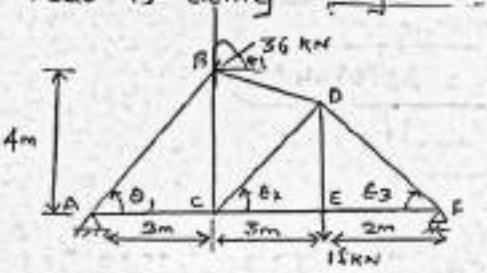
S.No	Member	Force	Nature
1	AC	16.517 kN	Compressive
2	AD	23.214 kN	Tensile
3	CD	10 kN	Tensile.
4	DB	23.214 kN	Tension
5	BC	-29.018	Compression.

Subject Code & Name**: 23AST205-Aerospace Structures** Date: **22.01.2025**

DAY: **05** TOPIC: ***Method of joints***

(12)

2. Find the force in member of truss, if 36 kN load is acting along the line of AB.



$\frac{36}{28.799} \sin\theta$

$\frac{28.799}{36} = \sin\theta$

Solution

To find θ_1, θ_2 and θ_3

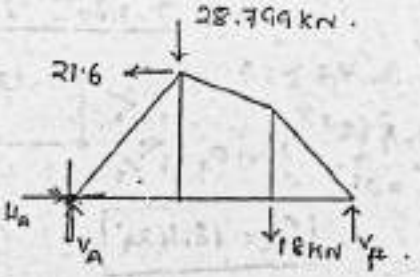
$\tan\theta_1 = 4/3 = 53.13^\circ$

$\tan\theta_2 = 3/3 = 45^\circ$

$\tan\theta_3 = 3/2 = 56.309^\circ$

To find Reactions H_A, V_A and V_F

Free Body Diagram



To find H_A, V_A and V_F.

$\Sigma F_y = 0$

$V_A + V_F = 18 + 28.799$

$\boxed{V_A + V_F = 46.799\text{ kN}}$

$\Sigma F_x = 0 \qquad H_A - 21.6 = 0$

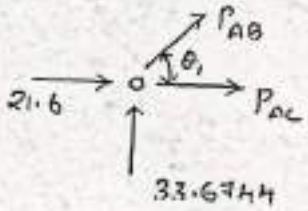
$\boxed{H_A = 21.6\text{ kN}}$

$\Sigma M_x = 0, \quad (21.6 \times 4) + V_F \times 8 - 18(6) - 28.799(3) = 0$

$$\boxed{V_F = 13.1246\ kN}$$

$$\boxed{V_A = 33.6744\ kN}$$

At Joint A



$\Sigma F_x = 0.$

$P_{AC} + P_{AB}\cos\theta_1 + 21.6 = 0.$

$P_{AC} + P_{AB}\cos\theta_1 = -21.6$

$$\boxed{P_{AC} = 3.654\ kN}$$

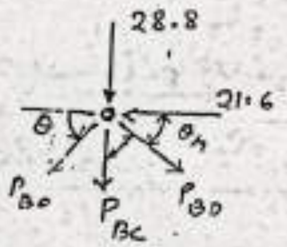
$\Sigma F_y = 0.$

$P_{AB}\sin\theta_1 + 33.6744 = 0.$

$P_{AB}\sin\theta_1 = -33.6744$

$$\boxed{P_{AB} = -42.09\ kN}$$

At Joint B



$90 - \theta_1 = 36.87. \Rightarrow \theta_1 \Rightarrow 53.13.$

$\tan\theta_4 = \frac{1}{3}$

$$\boxed{\theta_4 = 18.434}$$

$\Sigma F_x = 0.$

$-P_{BA}\cos\theta_1 - 21.6 + P_{BD}\cos\theta_4 = 0.$

$\Sigma F_y = 0$

$-28.8 - P_{BC} - P_{BA}\sin\theta_1 - P_{BD}\sin\theta_4 = 0$

$$-(-42.09)(\cos 53.13) - 21.6 + P_{BD}\cos(18.434) = 0.$$

$$P_{BD} = -\frac{3.6540}{\cos(18.434)}$$

$$\boxed{P_{BD} = -3.8516\ \text{kN}}$$

$$-28.8 - P_{BC} - (P_{BA}\sin\theta) - (P_{BD} * \sin\theta_4) = 0.$$

$$-28.8 - P_{BC} - (-42.09\sin(53.13)) + 3.8516\sin 18.434 = 0.$$

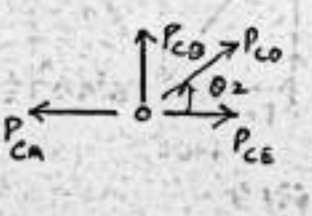
-42.09

$$-P_{BC} = -(42.09\sin(53.13) + 3.8516(0.3)) + 28.8$$

$$= -34.82743 + 28.8$$

$$\boxed{P_{BC} = 6.0274\ \text{kN}}$$

<u>At Joint C</u>



$\Sigma F_x = 0$

$\Sigma F_y = 0.$

$$\Rightarrow -P_{CA} + P_{CE} + P_{CD}\cos\theta_2 = 0.$$

$$\Rightarrow P_{CB} + P_{CD}\sin\theta_2 = 0.$$

$$6.0274 + P_{CD}\sin(45^\circ) = 0$$

$$P_{CD} = \frac{-6.0274}{\sin 45^\circ}$$

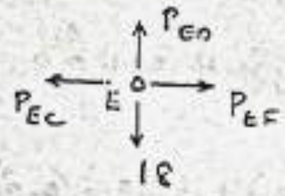
$$\boxed{P_{CD} = -8.5240\ \text{kN}}$$

$$-P_{AC} + P_{CE} + P_{CD}\cos\theta_2 = 0.$$

$$-3.654 + P_{CE} + (-8.5240)\cos 45^\circ = 0.$$

$$\boxed{P_{CE} = 9.6813\ kN}$$

Joint E

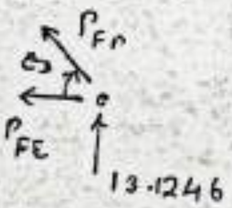


$$\boxed{P_{ED} = 18\ kN}$$

$$P_{EF} = P_{EC}$$

$$\boxed{P_{EF} = 9.6813\ kN}$$

JOINT F



$$\Sigma F_x = 0 \quad -P_{FE} - P_{FD}\cos\theta_3 = 0.$$

$$\Sigma F_y = 0$$

$$13.1246 + P_{FD}\sin\theta_3 = 0.$$

$$\Sigma F_x = 0 \quad -9.6813\ kN - P_{FD}\sin 56.309^\circ = 0$$

$$\Rightarrow \quad -P_{FD} = \frac{9.6813}{\sin(56.309)}$$

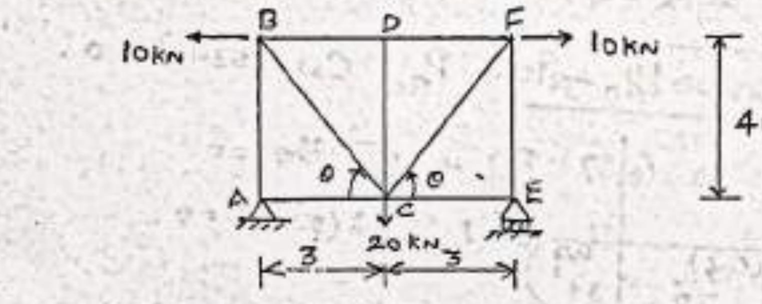
$$\boxed{P_{FD} = -15.9527\ kN}$$

Also $\boxed{P_{FD} = -15.7737\ kN.}$

RESULT

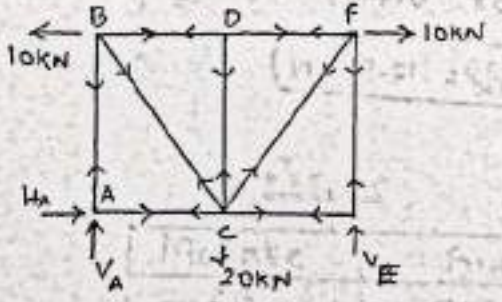
S.NO	MEMBER	FORCE (kN)	NATURE OF FORCE
1.	AC	3.654	Tension
2	AB	42.09	Compression
3	BD	3.8516	Compression
4	BC	6.0274	Tension
5	CD	8.5240	Compression
6	CE	9.6813	Tension
7	EF	9.6813	Tension
8	ED	18	Tension
9	FD	15.7722	Compression

3. Find the forces in members of truss shown below



Solution :

Free body Diagram



To find H_A, V_A and V_E

$\Sigma F_y = 0$

$V_A + V_E = 20\,kN$

$\Sigma F_x = 0$

$H_A = 10 - 10$

$\boxed{H_A = 0}$

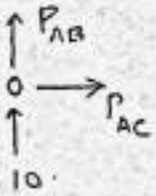
$\Sigma M_A = 0$

$(20 \times 3) + (10 \times 4) = (10 \times 4) + V_E(6)$

$60 + 40 = 40 + 6V_E$

$\boxed{V_A = 10\,kN}$ $\boxed{V_E = 10\,kN}$

Joint A

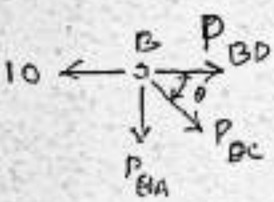


$\Sigma F_x = 0$

$P_{AC} = 0.$

$\boxed{P_{AB} = -10 KN}$

Joint. B



$\Sigma F_x = 0$ $\quad\quad \theta = 53.13$

$P_{BD} - 10 + P_{BC} \cos(53.13) = 0.$

$P_{BD} - 10 + 12.5 \times (0.6) = 0$

$\boxed{P_{BD} = 2.499 KN}$

$P_{BA} + P_{BC} \sin\theta = 0$

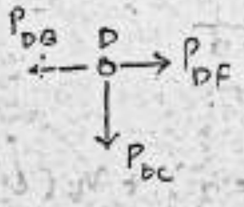
$P_{BA} + P_{BC} (\sin(53.13)) = 0.$

$-10 + P_{BC} (0.7999) = 0$

$P_{BC} = \frac{10}{0.7999}$

$\boxed{P_{BC} = 12.5 KN}$

JOINT- D



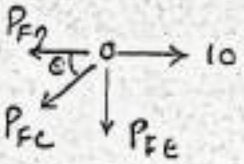
$\Sigma F_x = 0.$

$P_{DB} = P_{DF}$

$\boxed{P_{DF} = 2.499 KN}$

$\boxed{P_{DC} = 0}$

Joint - F



$\Sigma F_x = 0$ $\qquad \theta = 53.130$

$-P_{FC}\cos\theta - P_{FD} + 10 = 0.$

$\cancel{P_{FD} = 10\,kN}$

$\Sigma F_y = 0.$

$-(P_{FE} + P_{FC}\sin\theta) = 0$

$\Rightarrow \quad -P_{FC}\cos(53.13) - 2.499 + 10 = 0$

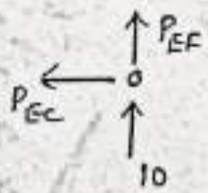
$-P_{FC}\cos(53.13) = -7.501$

$$\boxed{P_{FC} = 12.50\,kN}$$

$-(P_{FE} + P_{FC}\sin\theta) = 0.$

$$P_{FE} = -9.9999\,kN$$

Joint - E



$P_{EC} = 0.$

$P_{EF} = -10\,kN.$

Result

Member	Force	Nature
AB	-10kN	Compressive
AC	0	—
BD	2.499	Tensile
BC	12.5	Tensile
DC	0	—
DF	2.499	Tensile
CF	12.50	Tensile.
CE	0	
EF	-9.999	Compressive