Let W = Loss taking place in a machine in watts

G = Mass of the machine in kg

S = Specific heat in watt-sec/kg °C

θ = Rise in temperature above ambient temperature in °C

θ_F = Final temperature rise with continuos load in °C

A = Area of cooling surface in m^2

λ = Rate of heat dissipation in watts/sq meter/°C rise in temperature.

Let us consider the small time interval dt in which temperature rise of the machine is $d\theta$ due to the losses taking place in the machine. Total losses in machine during

time interval dt = W dt

Heat dissipation from surface during the same time interval = $A\lambda\theta \cdot dt$

Additional heat stored in the machine = $G.S.d\theta$

We have, heat developed = heat absorbed + heat dissipated

$$W \cdot dt = G.S.d\theta + A\lambda\theta \cdot dt \quad \text{...(i)}$$

$$\therefore \quad W \cdot dt - A\lambda\theta \cdot dt = G.S.d\theta$$

$$\therefore \quad (W - A\lambda\theta)\, dt = G.S.d\theta$$

$$\frac{dt}{G.S.} = \frac{d\theta}{W - A\lambda\theta}$$

$$\frac{dt}{\left(\frac{GS}{A\lambda}\right)} = \frac{d\theta}{\left(\frac{W}{A\lambda} - \theta\right)} \quad \ldots\text{(ii)}$$

When final temperature is reached, there is no heat absorbed. The heat which is generated is totally dissipated.

$$\therefore \quad W \cdot dt = A\,\lambda\,\theta_F\,dt$$

$$\therefore \quad W = A\,\lambda\,\theta_F$$

$$\therefore \quad \theta_F = \frac{W}{A\,\lambda} \quad \ldots\text{(iii)}$$

Substituting equation (iii) in equation (ii) we get,

$$\frac{dt}{\left(\frac{GS}{A\lambda}\right)} = \frac{d\theta}{\theta_F - \theta}$$

Integrating both sides of above equation

$$\int \frac{dt}{\left(\frac{GS}{A\lambda}\right)} = \int \frac{d\theta}{\theta_F - \theta}$$

$$\frac{A\,\lambda}{GS} \cdot t = -ln(\theta_F - \theta) + K \quad \ldots\text{(iv)}$$

where K = constant of integration

To find out value of K, let us use initial condition

At $t = 0, \quad \theta = \theta_1$

$$\therefore \quad 0 = -ln(\theta_F - \theta_1) + K$$

$$\therefore \quad K = ln(\theta_F - \theta_1)$$

Substituting this value of K in equation (iv),

$$\frac{A\,\lambda}{GS} \cdot t = -ln(\theta_F - \theta) + ln(\theta_F - \theta_1)$$

$$\therefore \quad \frac{A\,\lambda}{GS} \cdot t = ln\left(\frac{\theta_F - \theta_1}{\theta_F - \theta}\right)$$

$$\therefore \quad e^{\frac{A\,\lambda}{GS} t} = \frac{\theta_F - \theta_1}{\theta_F - \theta}$$

$$\therefore \quad \theta_F - \theta = (\theta_F - \theta_1)\,e^{-\frac{A\,\lambda}{GS} t}$$

$$\therefore \quad \theta = \theta_F - (\theta_F - \theta_1)\,e^{-\frac{A\,\lambda}{GS} t}$$

The term GS/A λ is called **heating time constant** of the machine and is denoted by τ.

$$\therefore \quad \boxed{\theta = \theta_F - (\theta_F - \theta_1)\,e^{-t/\tau}}$$

If the machine is started from ambient temperature $\theta_1 = 0°C$ then above equation becomes,

$$\therefore \quad \boxed{\theta = \theta_F \left(1 - e^{-t/\tau}\right)}$$

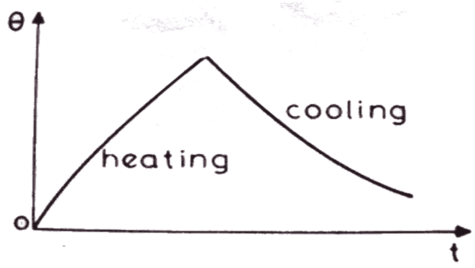
Let us consider time period $t = \tau$ then

$$\theta = \theta_F \left(1 - e^{-1}\right) = \theta_F \left(1 - \frac{1}{e}\right) = 0.632\ \theta_F$$

Similarly, at $t = 2\tau, \quad \theta = 0.865\ \theta_F$

$t = 3\tau, \quad \theta = 0.95\ \theta_F$

$t = 4\tau, \quad \theta = 0.982\ \theta_F$



Heating and cooling curves