Subject Code & Name**: 23AST205-Aerospace Structures** Date: **21.01.2025**

DAY: **02** TOPIC. ***PROPPED CANTILEVER BEAM***

Analysis of Statically Indeterminate Beams

The moment area method and the conjugate beam method can be easily applied for the analysis of statically indeterminate beams using the principle of superposition. Depending upon the degree of indeterminacy of the beam, designate the excessive reactions as redundant and modify the support. The redundant reactions are then treated as unknown forces. The redundant reactions should be such that they produce the compatible deformation at the original support along with the applied loads. For example consider a propped cantilever beam as shown in Figure 5.1(a). Let the reaction at *B* be *R* as shown in Figure 5.1(b) which can be obtained with the compability condition that the downward vertical deflection of *B* due to applied loading (i.e. Δ_0 shown in Figure 5.1(c)) should be equal to the upward vertical deflection of *B* due to *R* (i.e. Δ_0 shown in Figure 5.1(d)).

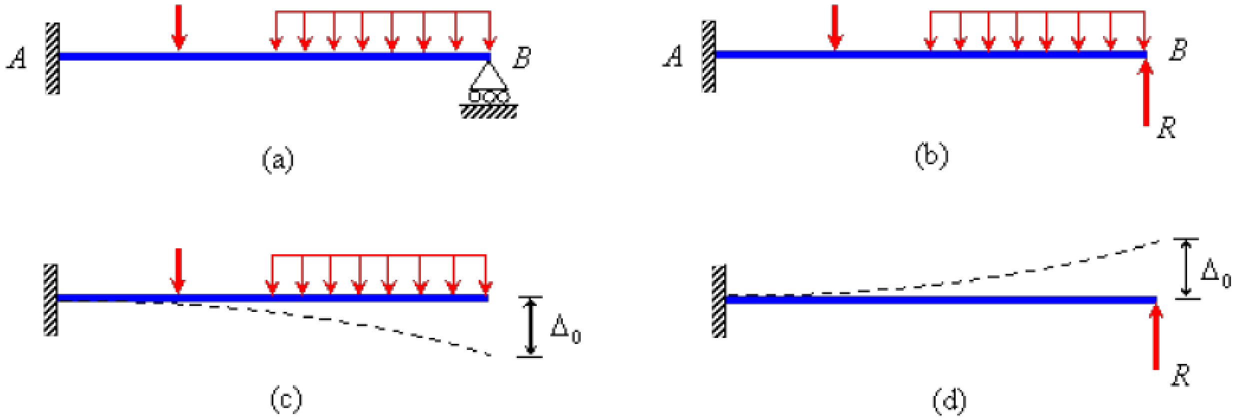

Figure 5.1

Example 5.1 Determine the support reactions of the propped cantilever beam as shown in Figure 5.2(a).

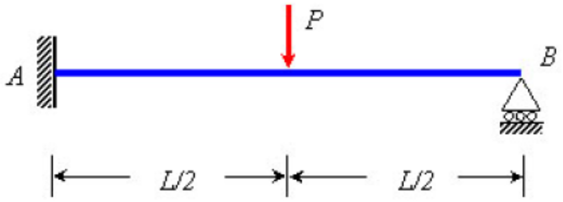

Figure 5.2(a)

Solution: The static indeterminacy of the beam is = 3 − 2 = 1. Let reaction at *B* is *R* acting in the upward direction as shown in Figure 5.2(b). The condition available is that the $\Delta_B = 0$.

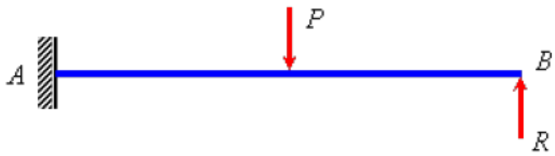

Figure 5.2(b)

(a) *Moment area method*

The bending moment diagrams divided by *EI* of the beam are shown due to *P* and *R* in Figures 5.2(c) and (d), respectively.

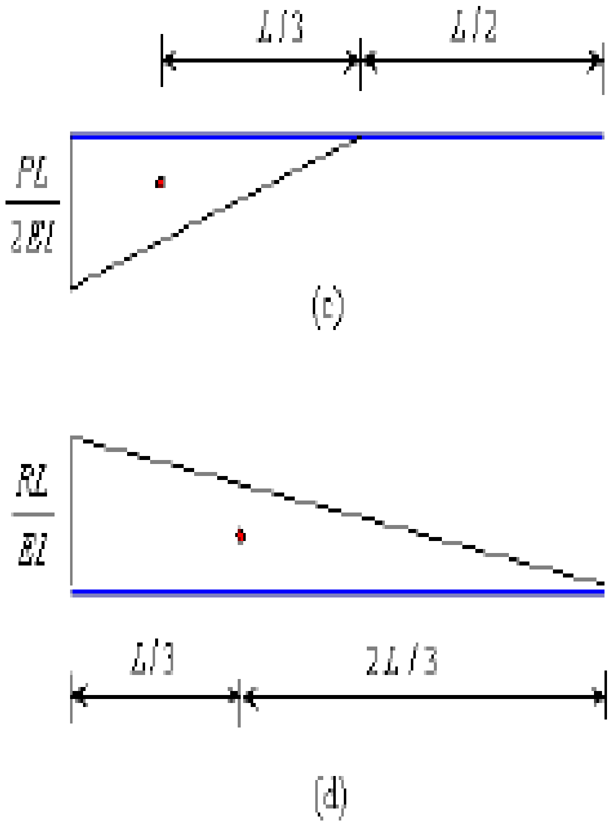

Figure 5.2(c-d)

Since in the actual beam the deflection of the point *B* is zero which implies that the deviation of point *B* from the tangent at *A* is zero. Thus,

$$t_{BA} = 0$$

or

$$-\frac{1}{2}\times\frac{L}{2}\times\frac{PL}{2EI}\left(\frac{L}{2}+\frac{L}{3}\right)+\frac{1}{2}\times L\times\frac{RL}{EI}\left(\frac{2L}{3}\right)=0$$

$$\therefore \quad R = \frac{5P}{16} \quad A_{m_i} = \frac{AE}{L}\left[(D_{m_3} - D_{m_1})C_x + (D_{m_4} - D_{m_2})C_y\right]$$

Taking moment about A , the moment at A is given by

$$M_A = P \times \frac{L}{2} - \frac{5P}{16} \times L = \frac{3PL}{16} \quad (\circlearrowright)$$

The vertical rection at A is

$$V_A = P - \frac{5P}{16} = \frac{11P}{16} \quad (\uparrow)$$

The bending moment diagram of the beam is shown in Figure 5.2(e).

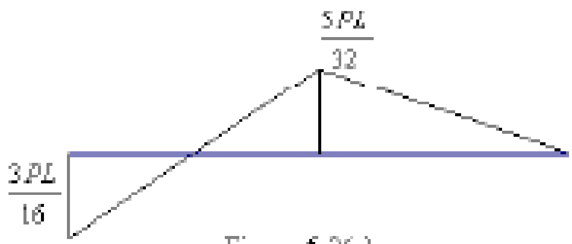

Figure 5.2(e)

(b) *Conjugate beam method*

The corresponding conjugate beam of the propped cantilever beam and loading acting on it are shown in Figure 5.2(f).

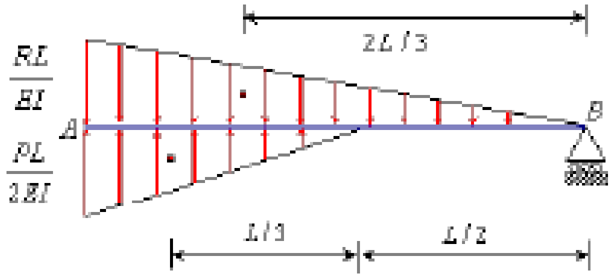

Figure 5.2(f)

The unknown R can be obtained by taking moment about B i.e.

$$-\frac{1}{2} \times \frac{L}{2} \times \frac{PL}{2EI}\left(\frac{L}{2} + \frac{L}{3}\right) + \frac{1}{2} \times L \times \frac{RL}{EI}\left(\frac{2L}{3}\right) = 0$$

$$\therefore \quad R = \frac{5P}{16}$$

Example 5.3 Determine the support reactions of the fixed beam with one end fixed and other supported on spring as shown in Figure 5.4(a). The stiffness of spring is $k = EI/L^3$.

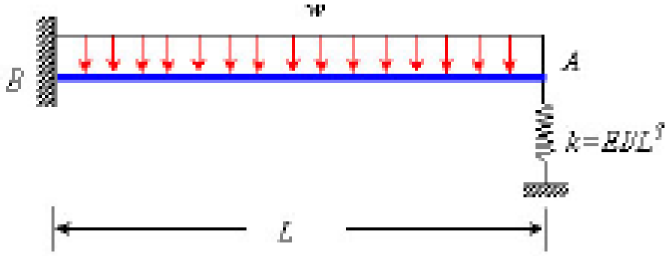

Figure 5.4(a)

Solution: The static indeterminacy of the beam is = 3−2 =1. Let the force in the spring be R . The free body diagram of the beam along with the *M/EI* diagram and spring are shown in Figure 5.4(b) and (c), respectively. The unknown R can be obtained with the condition that the vertical deflection of the free end of the beam and spring is identical.

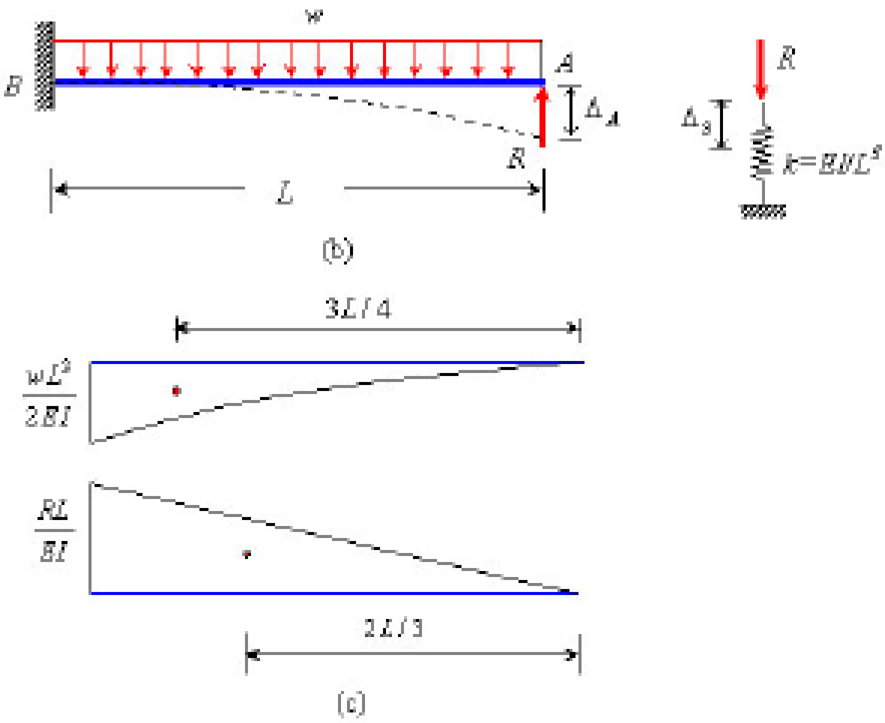

Figure 5.4(b)-(c)

Using moment area theorem, the deflection of free end A of the beam is

$$\Delta_A = -t_{A/B} = \left(\frac{1}{3}\times\frac{wL^2}{2EI}\times L\right)\times\frac{3L}{4}-\frac{1}{2}\times\frac{RL}{EI}\times L\times\frac{2L}{3}$$

$$\Delta_A = \frac{wL^4}{8EI}-\frac{RL^3}{3EI}$$

The downward deflection of spring is

$$\Delta_S = \frac{R}{k} = \frac{RL^3}{EI}$$

Equating Δ_A and Δ_S

$$\frac{RL^3}{EI} = \frac{wL^4}{8EI}-\frac{RL^3}{3EI}$$

$$\therefore \quad R = \frac{3wL}{32}$$

The bending moment at B

$$M_B = w\times L\times\frac{L}{2}-\frac{3wL}{32}\times L$$

$$= \frac{13wL^2}{32} \quad [\circlearrowleft]$$

The vertical reaction at B

$$V_B = w\times L-\frac{3wL}{32}$$

$$= \frac{29wL}{32} \quad [\uparrow]$$

The force in the spring $= \dfrac{3wL}{32}$ (compressive)

The bending moment diagram of the beam is shown in Figure 5.4(d).

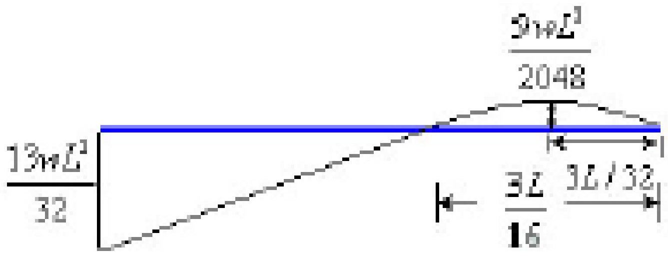

Figure 5.4(d)