DISCRETE RANDOM VARIABLE

DISCRETE

(1) DEFINITION:

A random variable x is discrete if it assumes only discrete values [finite (or) countably infinite]

(2) PROBABILITY MASS FUNCTION [PMF]

If x is a discrete random variable then the function $p(x) = p(X=x)$ is called p.m.f. of x provided satisfy the following conditions:

(i) $p(x_i) \geq 0 \quad , \forall \ i = 1, 2, 3, \ldots$

(ii) $\sum_{i=1}^{\infty} p(x_i) = 1$

(3) TO FIND CONSTANTS [k, a, c ...]

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

(4) CUMULATIVE DISTRIBUTION FUNCTION (or) DISTRIBUTION FUNCTIC

$$F(x) = p(X \le x) = \sum_{x_i \le x} p(x_i)$$

If cumulative distribution is given then to find p.m.f, $P[X = x_i] = F[x_i] - F[x_i - 1]$

(5) TO FIND MEAN (or) FIRST MOMENT :

$$E(X) = \mu_1' = \sum_{i=1}^{\infty} x_i \, p(x_i)$$

(6) TO FIND SECOND MOMENT :

$$E(X^2) = \mu_2' = \sum_{i=1}^{\infty} x_i^2 \, p(x_i)$$

(7) TO FIND VARIANCE:

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \sum_{i=1}^{\infty} x_i^2 p(x_i) - \left[\sum_{i=1}^{\infty} x_i \, p(x_i)\right]^2$$

(8) TO FIND MOMENTS :

$$\mu_1' = E(x) = \sum_{i=1}^{\infty} x_i p(x_i) \quad \text{, first moment}$$

$$\mu_2' = E(x^2) = \sum_{i=1}^{\infty} x_i^2 p(x_i) \quad \text{, second moment}$$

$$\mu_3' = E(x^3) = \quad \text{, Third moment}$$

$$\mu_4' = E(x^4) = \quad \text{, fourth moment.}$$

. . . .

$$\mu_r' = E(x^r) = \sum_{i=1}^{\infty} x_i^r p(x_i) \text{, } r^{th} \text{ moment .}$$

9) MOMENT GENERATING FUNCTION [MGF] :

$$M_x(t) = E[e^{tx}] = \sum_{x=0}^{\infty} e^{tx} p(x)$$

A random Variable x has the following probability function:

$X = x$	:	-2	-1	0	1	2	3
$p(x)$	:	0.1	k	0.2	$2k$	0.3	$3k$

(i) Find the value of k

(ii) Find $p(x<2)$; $p(-2<x<2)$; $p(0<x\leq 3)$; $p(-1\leq x\leq 3)$

(iii) Find the distribution function of x.

(iv) Find Mean & Variance.

(v) Find 3^{rd} moment.

(vi) Find Moment generating Function.

Soln:

(i) To Find k:

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

$$0.1+k+0.2+2k+0.3+3k=1$$

$$0.6+6k=1$$

$$6k=0.4$$

$$k=4/60$$

(i) $k = 1/15$

$X = x$	-2	-1	0	1	2	3
$P(x)$	$1/10$	$1/15$	$2/10$	$2/15$	$3/10$	$3/15$

(a) $p(x<2) = p(x=-2) + p(x=-1) + p(x=0) + p(x=1)$

$= 1/10 + 1/15 + 2/10 + 2/15$

$= 1/2$

(b) $P(-2<x<2) = p(x=-1) + p(x=0) + p(x=1)$

$= 1/15 + 2/10 + 2/15$

$= 2/5$

(c) $p(0<x\leq 3) = p(x=1) + p(x=2) + p(x=3)$

$= 2/15 + 3/10 + 3/15$

$= 19/30$

(d) $p(-1\leq x\leq 3) = p(x=-1) + p(x=0) + p(x=1) + p(x=2) + p(x=3)$

$= 1/15 + 2/10 + 2/15 + 3/10 + 3/15$

$= 9/10$

(iii) Distribution function of X:

X:	-2	-1	0	1	2	3
F(x):	1/10	1/6	11/30	1/2	4/5	1

X : x	$F(x) = p(X \le x)$
-2	$F(-2) = p(X \le -2) = 1/10$
-1	$F(-1) = p(X \le -1) = p(X=-2) + p(X=-1) = 1/10 + 1/15 = 1/6$
0	$F(0) = p(X \le 0) = p(X=-2) + p(X=-1) + p(X=0) = 11/30$
1	$F(1) = p(X \le 1) = p(X=-2) + p(X=-1) + p(X=0) + p(X=1) = 1/2$
2	$F(2) = p(X \le 2) = p(X=-2) + p(X=-1) + p(X=0) + p(X=1) + p(X=2) = 4/5$
3	$F(3) = p(X \le 3) = p(X=-2) + p(X=-1) + p(X=0) + p(X=1) + p(X=2) + p(X=3) = 1$

To find Mean & variance:

$$E(X) = \sum_i x_i \, p(x_i)$$

$$= (-2) \times 1/10 + (-1) \times 1/15 + 0 + 1 \times 2/15 + 2 \times 3/10 + 3 \times 3/15$$

$$= -\frac{1}{5} - \frac{1}{15} + \frac{2}{15} + \frac{3}{5} + \frac{3}{5}$$

$$= 16/15$$

$$E(X^2) = \sum_i x_i^2 p(x_i)$$

$$= (-2)^2 \times 1/10 + (-1)^2 \times 1/15 + 0 + 1 \times 2/15 + (2)^2 \times 3/10 + (3)^2 \times 3/15$$

$$= \frac{4}{10} + \frac{1}{15} + \frac{2}{15} + \frac{12}{10} + \frac{27}{15}$$

$$= 18/5$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= 18/5 - (16/15)^2$$

(v) To find 3rd moment:

$$E(X^3) = \sum_i x_i^3 p(x_i)$$

$$= (-2)^3 \times 1/10 + (-1)^3 \times 1/15 + 0 + 2/15 + (2)^3 \times 3/10 + (3)^3 \times 3/15$$

$$= -\frac{4}{5} - \frac{1}{15} + \frac{2}{15} + \frac{12}{5} + \frac{27}{5} = \frac{106}{15}$$

(vi) To find Moment generating function:

$$M_X(t) = E(e^{tx}) = \sum_{x=-2}^{3} e^{tx} p(x)$$

$$= e^{-2t}(1/10) + e^{-t}(1/15) + 2/10 + e^{t}(2/15) + e^{2t}(3/10) + e^{3t}(3/15)$$

A discrete random variable x has the probability function given below:

x:	0	1	2	3	4	5	6	7
$p(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) the value of k.

(ii) $p(x<6)$, $p(x \geq 6)$, $p(0<x<4)$

(iii) Distribution function of x

(iv) Find the smallest value of x, if $p(x \leq x) > 1/2$

(v) Find the probability function of $Y = 2x+5$

(vi) $P(1/2 < x < 5/2 \,/\, x > 1)$

Soln:

(i) WKT $\sum_{i=1}^{\infty} p(x_i) = 1$

$$p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)+p(x=5)+p(x=6)+p(x=7) = 1$$

$$\Rightarrow \quad 0+k+2k+2k+3k+k^2+2k^2+7k^2+k = 1$$

$$\Rightarrow \quad 10k^2+9k-1 = 0$$

$\Rightarrow 10k^2+9k-1=0$

$\Rightarrow 10k^2+10k-k-1=0$

$\Rightarrow 10k(k+1)-(k+1)=0$

$\Rightarrow (k+1)(10k-1)=0$

$\Rightarrow k=-1\ ;\ k=1/10$

$k=-1$ is impossible. we choose $k=1/10$ since probability value ≥ 0.

$\therefore\ x:$	0	1	2	3	4	5	6	7
$p(x):$	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$2/100$	$17/100$

(ii) (a) $p(x<6) = p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)+p(x=5)$

$$= 0 + 1/10 + 2/10 + 2/10 + 3/10 + 1/100$$

$$= \frac{10+20+20+30+1}{100}$$

$$= \frac{81}{100}$$

(b) $p(x\geq 6) = 1-p(x<6)$

$$= 1-\frac{81}{100}$$

$$= \frac{19}{100}$$

(c) $p(0<x<4) = p(x=1)+p(x=2)+p(x=3)$

$= 1/10 + 2/10 + 2/10 = 5/10$

(iii) Distribution function of x:

x	0	1	2	3	4	5	6	7
$F(x)$	0	$1/10$	$3/10$	$5/10$	$8/10$	$81/100$	$83/100$	$100/100 = 1$

(iv) The smallest value of x, if $p(x \le x) > 1/2$

x	0	1	2	3	4	5	6	7
$F(x)$	0	0.1	0.3	0.5	0.8	0.81	0.83	1

Since $p(x \le x) > 1/2 = 0.5$, this is true for $x = 4, 5, 6, 7$

the smallest value of x is 4. $\Rightarrow x = 4$.

(v) Probability function of $Y = 2x+5$

$X = x$	0	1	2	3	4	5	6	7
$Y = 2x+5$	5	7	9	11	13	15	17	19

y :	5	7	9	11	13	15	17	19
$p(y)$:	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$2/100$	$17/100$

(vi) $P(1/2 < x < 5/2 \,/\, x > 1)$

WKT $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{P((1/2 < x < 5/2) \cap (x > 1))}{P(x > 1)}$$

$$= \frac{P[(0.5 < x < 2.5) \cap (x > 1)]}{P(x > 1)}$$

$$= \frac{P([(x=1),(x=2)] \cap [(x=2),(x=3),(x=4),(x=5),(x=6),(x=7)])}{P[(x=2),(x=3),(x=4),(x=5),(x=6),(x=7)]}$$

$$= \frac{P(x=2)}{P((x=2),(x=3),(x=4),(x=5),(x=6),(x=7))}$$

$$= \frac{2/10}{2/10 + 2/10 + 3/10 + 1/100 + 2/100 + 17/100} = \frac{2/10}{90/100} = 2/9$$