



## DEPARTMENT OF MATHEMATICS

### UNIT – I PROBABILITY AND RANDOM VARIABLES

#### CONTINUOUS RANDOM VARIABLE

The density function of a random variable  $x$  is given by  $f(x) = kx(2-x)$ ,  $0 \leq x < 2$ , Find  $k$ , mean, variance and 3<sup>rd</sup> moment.

Soln:

(i) to find  $k$ :

$$\text{Wkt } \int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_0^2 kx(2-x) dx = 1$$

$$\Rightarrow k \int_0^2 (2x - x^2) dx = 1$$

$$\Rightarrow k \left[ x^2 - \frac{x^3}{3} \right]_0^2 = 1$$

$$\Rightarrow k \left[ 4 - \frac{8}{3} \right] = 1$$

$$\Rightarrow k = \frac{3}{4}$$

$$\therefore f(x) = \frac{3}{4} x(2-x)$$



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(ii) To find mean :

$$\begin{aligned}\text{WKT } E(x) &= \int_{-\infty}^{\infty} x f(x) dx = \int_0^2 x \cdot \frac{3}{4} x(2-x) dx = \frac{3}{4} \int_0^2 (2x^2 - x^3) dx \\ &= \frac{3}{4} \left[ \frac{2x^3}{3} - \frac{x^4}{4} \right]_0^2 = 1\end{aligned}$$

(iii) To find Variance :

$$\begin{aligned}\text{Now } E(x^2) &= \int_{-\infty}^{\infty} x^2 f(x) dx = \int_0^2 x^2 \cdot \frac{3}{4} x(2-x) dx = \frac{3}{4} \int_0^2 (2x^3 - x^4) dx \\ &= \frac{3}{4} \left[ \frac{2x^4}{4} - \frac{x^5}{5} \right]_0^2 = \frac{6}{5}\end{aligned}$$

$$\therefore \text{Var}(x) = E(x^2) - (E(x))^2 = \frac{6}{5} - 1 = \frac{1}{5}$$

(iv) To find 3<sup>rd</sup> moment :

$$\begin{aligned}E(x^3) &= \int_{-\infty}^{\infty} x^3 f(x) dx = \int_0^2 x^3 \cdot \frac{3}{4} x(2-x) dx = \frac{3}{4} \int_0^2 (2x^4 - x^5) dx \\ &= \frac{3}{4} \left[ \frac{2x^5}{5} - \frac{x^6}{6} \right]_0^2 = \frac{8}{5}\end{aligned}$$



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The p.d.f of abs. R.V  $x$  is  $f(x) = ke^{-|x|}$ . Find  $k$  &  $F(x)$

Soln:  
(i) WKT  $\int_{-\infty}^{\infty} f(x) dx = 1$

$$\int_{-\infty}^{\infty} ke^{-|x|} dx = 1$$

$$2k \int_0^{\infty} e^{-x} dx = 1$$

$$2k \left[ \frac{e^{-x}}{-1} \right]_0^{\infty} = 1$$

$$2k = 1 \Rightarrow k = \frac{1}{2}$$

$$\therefore f(x) = \frac{1}{2} e^{-|x|}$$

$$\text{Now } f(x) = \frac{1}{2} e^{-|x|} = \begin{cases} \frac{1}{2} e^{-(-x)}, & -\infty < x < 0 \\ \frac{1}{2} e^{-(x)}, & 0 < x < \infty \end{cases}$$
$$= \begin{cases} \frac{1}{2} e^x, & -\infty < x < 0 \\ \frac{1}{2} e^{-x}, & 0 < x < \infty \end{cases}$$

(ii) For  $x \leq 0$ ,

$$F(x) = \int_{-\infty}^x f(x) dx = \int_{-\infty}^x \frac{1}{2} e^x dx = \left[ \frac{e^x}{2} \right]_{-\infty}^x = \frac{e^x}{2}$$



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For  $x > 0$

$$\begin{aligned} F(x) &= \int_{-\infty}^0 f(x) dx + \int_0^x f(x) dx \\ &= \int_{-\infty}^0 \frac{1}{2} e^x dx + \int_0^x \frac{1}{2} e^{-x} dx = \frac{1}{2} e^x \Big|_{-\infty}^0 + \frac{1}{2} \frac{e^{-x}}{-1} \Big|_0^x \\ &= \frac{1}{2} + \frac{1}{2} [e^{-x} - 1] \\ &= \frac{1}{2} [2 - e^{-x}] \end{aligned}$$

A continuous random variable  $x$  has the distribution function  $F(x) = \begin{cases} 0 & , x \leq 1 \\ k(x-1)^2 & , 1 < x \leq 3 \\ 0 & , x > 3 \end{cases}$ . Find pdf;  $f(x)$ ,  $k$ , and  $P[x < 2]$ .



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Soln:

(i) If  $F(x)$  given then to find  $f(x)$ , wkt

$$f(x) = \frac{d}{dx} [F(x)]$$

$$= \frac{d}{dx} [k(x-1)^4] = 4k(x-1)^3$$

$$\therefore f(x) = \begin{cases} 0, & x \leq 1 \\ 4k(x-1)^3, & 1 < x \leq 3 \\ 0, & x > 3 \end{cases}$$

(ii) To find  $k$ ,

$$\text{wkt } \int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_{-\infty}^1 f(x) dx + \int_1^3 f(x) dx + \int_3^{\infty} f(x) dx = 1$$

$$\int_1^3 4k(x-1)^3 dx = 1$$

$$4k \left[ \frac{(x-1)^4}{4} \right]_1^3 = 1$$

$$\Rightarrow k(2^4) - k(0) = 1$$

$$\Rightarrow 16k = 1 \Rightarrow k = \frac{1}{16}$$



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$$\therefore f(x) = \begin{cases} 0 & , x \leq 1 \\ \frac{1}{4} (x-1)^3, & 1 < x \leq 3 \\ 0 & , x > 3 \end{cases}$$

(iii)  $P[X < 2]$ :

$$\text{WKT } P[X \leq x] = F[x]$$

$$P[X < 2] = F[2]$$

$$= \frac{1}{4} [2-1]^3 = \frac{1}{4}$$