PROPERTIES

MATHEMATICAL EXPECTATIONS :

Let 'x' be a random variable with probability density function $f(x)$, or probability mass function $p(x)$ then the mathematical expectation of 'x' is denoted by $E(x)$ and is given by

$$E(x) = \sum_{x} x p(x), \text{ for a discrete random Variable}$$
$$= \int_{-\infty}^{\infty} x f(x)\, dx, \text{ for a Continuous random Var}$$

The variance of a random variable 'x' is denoted by var(x) and is defined by

$$var(x) = E(x^2) - (E(x))^2$$
$$= \sum_{x} x^2 p(x) - \left[\sum_{x} x p(x)\right]^2, \text{ for a discrete RV.}$$
$$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - \left[\int_{-\infty}^{\infty} x f(x)\, dx\right]^2, \text{ for a cts. RV.}$$

PROPERTIES :

If 'x' and 'y' are random variable and a, b are constants, then

(i) $E(a) = a$

(ii) $E(ax) = a E(x)$

(iii) $E(ax+b) = a E(x) + b$

(iv) If $y \leq x$, then $E(y) \leq E(x)$

(v) If x & y are independent then $E(xy) = E(x) \cdot E(y)$

(vi) $E(x^2) \geq (E(x))^2$

(vii) $Var(x) \geq 0$

(viii) $Var(a) = 0$

(ix) $Var(ax) = a^2 var(x)$

(x) $Var(x \pm a) = var(x)$

(xi) $Var(ax+b) = a^2 Var(x)$

(xii) $Var(ax+by) = a^2 var(x) + b^2 var(y)$

PROBLEMS:

(1) When a die is thrown, 'x' denotes the number that turns up. Find E(x), E(x²) and var(x).

Soln:

Let 'x' denotes the number that turns up in die.

(i.e.) 'x' takes values 1, 2, 3, 4, 5, 6.

x:	1	2	3	4	5	6
$p(x)$:	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

Now $E(x) = \sum_x x\,p(x)$

$= 1\times 1/6 + 2\times 1/6 + 3\times 1/6 + 4\times 1/6 + 5\times 1/6 + 6\times 1/6$

$= 21/6 = 7/2$

$E(x^2) = \sum_x x^2 p(x)$

$= (1)^2\times 1/6 + (2)^2\times 1/6 + (3)^2\times 1/6 + (4)^2\times 1/6 + (5)^2\times 1/6 + (6)^2\times 1/6$

$= 91/6$

$Var(x) = E(x^2) - (E(x))^2$

$= 91/6 - (21/6)^2$

$= \frac{546-441}{36} = \frac{105}{36}$

$= \frac{35}{12}$

Find $E(x^2)$ and $E(2x+3)$ for the following probability distribution:

x:	-2	-1	0	1	2	3
$p(x)$:	$1/6$	$1/6$	0	$2/3$	$1/6$	$1/6$

Soln:

Now $E(x^2) = \sum_x x^2 p(x)$

$$= (-2)^2 \times 1/6 + (-1)^2 \times 1/6 + 0 + (1)^2 \times 2/6 + (2)^2 \times 1/6 + (3)^2 \times 1/6$$

$$= \frac{4+1+4+4+9}{6} = \frac{22}{6} = \frac{11}{3}$$

$$E(2x+3) = 2E(x) + 3$$

Now $E(x) = \sum_x x\,p(x)$

$$= (-2) \times 1/6 + (-1) \times 1/6 + 0 + 1 \times 2/3 + 2 \times 1/6 + 3 \times 1/6$$

$$= \frac{-2-1+4+2+3}{6} = 1$$

$$\therefore E(2x+3) = 2(1) + 3$$

$$= 5$$

Let x be a random variable with $E(x)=1$ and $E(x(x-1))=4$. Find $var(x)$ and $var(2-3x)$.

Soln:

$E(x)=1$; $E(x(x-1))=4$ (given)

Now $E(x(x-1))=4$

$E(x^2-x)=4$

$E(x^2)-E(x)=4$

$E(x^2)-1=4$

$\Rightarrow E(x^2)=5$

$\therefore var(x) = E(x^2)-(E(x))^2 = 5-1=4$

Now $Var(2-3x) = (-3)^2 var(x) = 9 \times (4) = 36$.