TEST FOR SINGLE PROPORTION:

If x is the no. of person or items possessing a certain attributes in a sample of 'n' items or person then the sample proportion, $p' = \frac{x}{n}$.

Null hypothesis, $H_0 : P = P_0$ where 'p' is population proportion

$H_1 : P \neq P_0$

Test statistic, $z = \frac{p' - P}{\sqrt{\frac{PQ}{n}}}$ where $q = 1 - p$.

1) A coin is tossed 256 times and 132 heads are obtained. would you conclude that the coin is a biased one?

Soln: Given: $n = 256$,

$x = 132$, no. of heads

population prop. of getting head: $P = \frac{1}{2}$ & $p' = \frac{x}{n} = \frac{132}{256} = 0.5156$ & $q = 1 - P = \frac{1}{2}$

Step 1: Formulating H_0 and H_1:

H_0: The coin is unbiased one (ie) $H_0 : P = \frac{1}{2}$

H_1: The coin is biased one (ie) $H_1 : P \neq \frac{1}{2}$

Step 2: LOS, $\alpha = 5\% = 0.05$.

Step 3 : Test Statistic, $z = \dfrac{p' - P}{\sqrt{\frac{PQ}{n}}}$

$$= \frac{0.5156 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{256}}}$$

$$= 0.4992.$$

Step 4 : Critical value at 5% (two tailed test) is

$$z_\alpha = 1.96$$

Step 5 : Conclusion : $z = 0.4992 < 1.96 = z_\alpha$

$\therefore$ H_0 is accepted at 5% LOS

$\therefore$ The coin is unbiased one.

★ In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?

[$z = 2.06$, $H_1 : P > 0.5$ Majority of men in this city are smokers]

2) Twenty people were affected by cholera and out of them only eighteen survived. Would you reject the hypothesis that the survival rate, if affected by cholera is 85% in favour of the hypothesis that it is more at 5% LOS.

Soln: Given : $n = 20$, $P = 85\%$ (ie) $P = \frac{85}{100} = 0.85$

$x = 18$ $\quad p' = \frac{18}{20} = 0.9$ $\quad$ & $Q = 1 - P = 0.15$

Step 1: Formulating H_0 and H_1

$H_0 : P = 0.85$, (ie) pepoles survived after attack

$H_1 : P > 0.85$ (one tailed test)

Step 2: LOS $\alpha = 5\% = 0.05$

Step 3: Test statistic, $Z = \frac{p' - P}{\sqrt{PQ/n}}$

$= \frac{0.9 - 0.85}{\sqrt{\frac{0.85 \times 0.15}{20}}}$

$= 0.626$

Step 4: Critical value at 5% (one tailed test) is

$Z_\alpha = 1.645$

step 5: Conclusion: $z = 0.626 < 1.645 = z_\alpha$

$\therefore$ H_0 is accepted at 5% LOS.

3) Experience has shown that 20% of a manufacture product is of top quality. In one day production of 400 articles only 50 are of top quality. Show that either the production of the day taken was not a representatives sample or the hypothesis of 20% was wrong.

Soln: Given: $n = 400$, $x = 50$, $p' = \frac{50}{400} = 0.125$

$p = 20\% = \frac{2}{100} = 0.20$, $q = 1 - P = 0.80$

step 1: Formulating H_0 and H_1

$H_0 : p = 0.20$

$H_1 : p \neq 0.20$ (two tailed test)

step 2: LOS $\alpha = 5\%$

step 3: Test statistic, $z = \frac{p' - p}{\sqrt{pq/n}}$

$= \frac{0.125 - 0.20}{\sqrt{\frac{0.20 \times 0.80}{400}}}$

$= -3.75$ (ie) $z = 3.75$

step 4: critical value at 5% (two tailed test) is

$z_\alpha = 1.96$

step 5: Conclusion: $z = 3.75 > 1.96 = z_\alpha$

$\therefore$ H_0 is rejected at 5% LOS

$\therefore$ The ~~day~~ production of the day taken was not a representative sample or the hypothesis of 20% was wrong.

4) In a sample of 500 peoples in kerala 280 are tea drinkers, othe rest are coffee drinkers. It can be assume that both coffee and tea are equally popular in the state at 5% LOS.

Soln: Given: $n = 500$, $x = 280$, $p' = \frac{x}{n} = \frac{280}{500} = 0.56$

$p = \frac{1}{2}$ That is, population proportion of tea drinkers & $q = 1 - p = 1/2$

Step 1: Formulating H_0 & H_1

$H_0 : p = \frac{1}{2}$

$H_1 : p \neq \frac{1}{2}$ (two tailed test)

step 2: LOS at 5% (ie) $\alpha = 0.05$

step 3: Test statistic, $z = \frac{p' - p}{\sqrt{pq/n}} = \frac{0.56 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{500}}} = 2.68$

step 4: critical value at 5% (two tailed test) is

$z_\alpha = 1.96$

Step 5: Conclusion: $z = 2.68 > 1.96 = z_\alpha$

$\therefore$ H_0 is rejected at 5% LOS.

$\therefore$ tea & coffee are not equally popular in the state.

TEST FOR DIFFERENCE OF PROPORTIONS:

Null hypothesis, $H_0: P_1 = P_2$.

Test statistic, $z = \dfrac{p_1' - p_2'}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ where $p_1' = \frac{x_1}{n_1}$ & $p_2' = \frac{x}{n}$

and $p = \dfrac{p_1'n_1 + p_2'n_2}{n_1+n_2} = \dfrac{x_1+x_2}{n_1+n_2}$ & $q = 1-p$.

1) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal, are same against that they are not, at 5% level.

Soln: Given: $n_1 = 400$, men, $x_1 = 200$

$n_2 = 600$, women, $x_2 = 325$

$$p_1' = \frac{x_1}{n_1} = \frac{200}{400} = 0.5 \quad \& \quad p_2' = \frac{x_2}{n_2} = \frac{325}{600} = 0.541$$

$$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{200+325}{400+600} = 0.525 \quad \& \quad q = 1 - P = 0.475$$

step 1: Formulating H_0 and H_1

$H_0: P_1 = P_2$, in favour of proposal (no diff. btw them)

$H_1: P_1 \neq P_2$ (two tailed test)

step 2: LOS $\alpha = 5\% = 0.05$

step 3 : Test statistic, $z = \dfrac{p_1' - p_2'}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

$$= \frac{0.5 - 0.541}{\sqrt{0.525 \times 0.475\left(\frac{1}{400}+\frac{1}{600}\right)}}$$

$$= \frac{-0.041}{\sqrt{0.001039}}$$

$$= -1.269$$

$$|z| = 1.269$$

step 4 : critical value at 5% LOS is $z_\alpha = 1.96$.

step 5 : Conclusion : $z = 1.269 < 1.96 = z_\alpha$

∴ H_0 is accepted to 5% LOS.

∴ the men & women do not differ significantly, as regards proposal of flyover is concerned.

2) In a large city A, 20% of a random sample of 900 school children had defective eye-sight. In other large city B, 15% of random sample of 1600 children had the same defect. Is this difference between the two proportions significant? [Obtain 95% confidence limits for the difference in the population proportions.]

Soln:
Given : In city A, $n_1 = 900$, $p_1' = 20\% = 0.20$

In city B, $n_2 = 1600$, $p_2' = 15\% = 0.15$

$$p = \frac{p_1' n_1 + p_2 n_2}{n_1 + n_2} = \frac{0.20(900) + 0.15(1600)}{900 + 1600}$$

$$= 0.168$$

$$q = 1 - p = 1 - 0.168 = 0.832$$

Step 1: Formulating H_0 and H_1.

$$H_0 : p_1 = p_2$$

$$H_1 : p_1 \neq p_2 \quad \text{(two tailed test)}$$

Step 2: LOS at $\alpha = 5\% = 0.05$

Step 3 : Test statistic, $z = \dfrac{p_1' - p_2'}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$$= \frac{0.20 - 0.15}{\sqrt{0.168 \times 0.832\left(\frac{1}{900} + \frac{1}{1600}\right)}}$$

$$= \frac{0.05}{0.0156}$$

$$z = 3.21$$

Step 4 : Critical value at 5% LOS is $z_\alpha = 1.96$

Step 5 : Conclusion: $z = 3.21 > 1.96 = z_\alpha$

$\therefore$ H_0 is rejected at 5% Los.

$\therefore$ The difference between the two proportions is significant.

[Confidence limit :

difference $p_1' - p_2' =$

$$(p_1' - p_2') \pm 1.96 \sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= (0.20 - 0.15) \pm 1.96\,(0.156)$$

$$= 0.05 \pm 0.031$$

$$= 0.019,\ 0.081]$$