TEST OF SIGNIFICANCE OF LARGE SAMPLES :

Tabulated values :

TEST FOR SINGLE MEAN :

Null Hypothesis, $H_0 : \mu = \mu_0$

Test statistics, $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ (or) $z = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

1) A sample of 900 members is found to have a mean of 3.4 cm and S.D. 2.61 cms. Is the sample from a large population of mean 3.25 cm and S.D. 2.61 cms. If the population is normal and its mean is unknown find the 95% confidential (fiducial) limits of true mean.

Soln: Given: $n = 900$, $\bar{x} = 3.4$, $\mu = 3.25$, $\sigma = 2.61$

Step 1: Formulating H_0 & H_1:

$H_0 : \mu = 3.25$

$H_1 : \mu \neq 3.25$ (two tailed test)

Step 2: Level of significance $\alpha = 5\% = 0.05$

Step 3 : Test statistic, $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$$= \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}}$$

$$= 1.724$$

Step 4 : Critical value at 5% is $z_\alpha = 1.96$.

Step 5 : Conclusion: Since $|z| = 1.724 < 1.96 = z_\alpha$,

Ho is accepted at 5% level of significance.

$\therefore$ The sample is taken from population whose mean is 3.25 cm.

Confidence limits:

$$\mu = \bar{x} \pm z_\alpha \frac{\sigma}{\sqrt{n}}$$

$$= 3.4 \pm 1.96 \times \frac{2.61}{\sqrt{900}} = 3.4 \pm 0.17$$

$$= 3.23,\ 3.57$$

(ie) $3.23 < \mu < 3.57$.

2) A random sample of 200 employees at a large corporation showed their average to be 42.8 years with a S.D of 6.89 years. Test the hypothesis $H_0: \mu = 40$, $H_1: \mu > 40$ at $\alpha = 0.01$ level of significance

Soln:

Given: $n = 200$, $\bar{x} = 42.8$, $\mu = 40$, $\sigma = 6.89$

Step 1: Formulating H_0 and H_1:

$H_0: \mu = 40$

$H_1: \mu > 40$ (one tail test - right)

Step 2: Level of significance, $\alpha = 0.01$.

Step 3: Test statistic, $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \dfrac{42.8 - 40}{\frac{6.89}{\sqrt{200}}}$

$= 5.747$

Step 4: Critical value at 1% (one tailed - right) is $Z_\alpha = 2.33$

Step 5: Conclusion: Since $|Z| = 5.747 > 2.33 = z_\alpha$

$\therefore$ H_0 is rejected at 1% level of significance

$\therefore$ the hypothesis, $H_1 : \mu > 40$ is accepted.

3) The mean height of college students in a city are normally distributed with S.D. 6 cms. A sample of 100 students has mean height of 158 cms. Test the hypothesis that the mean height of college students in the city 160 cms. Also obtain 99% confidence limits for the true mean.

Soln: Given: $n = 100$, $\bar{x} = 158$, $\mu = 160$, $\sigma = 6$

Step 1: Formulating H_0 and H_1:

$H_0 : \mu = 160$

$H_1 : \mu \neq 160$ (two tailed test)

Step 2: Level of significance, $\alpha = 1\%$

Step 3: Test statistic, $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$$= \frac{158 - 160}{6/\sqrt{100}}$$

$$= 3.33$$

Step 4: critical value at 1% (two side test) is

$$z_\alpha = 2.58$$

Step 5: Conclusion; Since $|z| = 3.33 > 2.58 = z_\alpha$

$\therefore$ H_0 is rejected at 1% level of significance.

$\therefore$ The mean height of the college students in the city is 160 cms is not true.

Confidence limit:

$$\mu = \bar{x} \pm z_\alpha \frac{\sigma}{\sqrt{n}}$$

$$= 158 \pm 2.58 \times \frac{6}{\sqrt{100}}$$

$$= 158 \pm 1.548$$

$$= 156.452,\ 159.548$$

(ie) $156.452 < \mu < 159.548$, here $\mu = 160$ does not lies in the interval.

TEST FOR DIFFERENCE FOR TWO MEANS:

Null hypothesis : $H_0 : \mu_1 = \mu_2$

test statistic, $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ $\sigma_1 = \sigma_2 = \sigma$

$$= \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

(or) $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

1) The means of two simple large samples of 1000 and 2000 members are 67.5 inches and 68 inches resp. Can the samples be regarded as drawn from the same population of standard deviation of 2.5 inches? Test at 5% level of significance (LOS)

Soln:

Given : $n_1 = 1000$, $\bar{x}_1 = 67.5$,

$n_2 = 2000$, $\bar{x}_2 = 68$, & $\sigma = 2.5$

Step 1: Formulating H_0 and H_1:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (two tailed test)

Step 2: Level of significance, $\alpha = 5\% = 0.05$

Step 3: Test statistic, $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{67.5 - 68}{2.5\sqrt{\frac{1}{1000} + \frac{1}{2000}}}$$

$$= -5.164$$

$$|z| = |-5.164| = 5.164$$

Step 4: critical value at 5% (two sided test) is $z_\alpha = 1.96$.

Step 5: Conclusion; $z = 5.164 > 1.96 = z_\alpha$

$\therefore$ H_0 is rejected at 5% LOS.

$\therefore$ The samples cannot be regarded as drawn from the same population of S.D. 25 inches.

2) A simple sample of height of 6400 sailors has a mean of 67.85 inches and s.D. of 2.56 inches while a simple sample of heights of 1600 soldiers has a mean of 68.55 inches and s.D of 2.52 inches. Do the data, indicate that soldiers are on the average taller than sailors? Use 5% LOS.

Soln:

Given: Sailors: $n_1 = 6400$, $\bar{x}_1 = 67.85$, $s_1 = 2.56$

Soldiers: $n_2 = 1600$, $\bar{x}_2 = 68.55$, $s_2 = 2.52$.

Step 1: Formulating H_0 and H_1

$H_0: \mu_1 = \mu_2$.

$H_1: \mu_1 < \mu_2$ (one tailed test - left)

Step 2: LOS at 5% (a) $\alpha = 0.05$

Step 3: Test Statistic,

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{67.85 - 68.55}{\sqrt{\frac{(2.56)^2}{6400} + \frac{(2.52)^2}{1600}}}$$

$$= -9.91$$

$$|z| = |-9.91|$$

$$= 9.91$$

Step 4: Critical value at 5% (one tail test) is $z_\alpha = 1.645$

step 5: Conclusion: $z = 9.91 > 1.645 = z_\alpha$

$\therefore$ Ho is rejected at 5% of LOS

$\therefore$ The data indicates that soldiers are on the average taller than sailors.

* A simple sample of heights of 6400 English men has a mean of 170 cm & S.D of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm & S.D. of 6.3 cm. Do the data indicate that Americans are the avg. taller than the English men? [$z = 11.32$, $\mu_1 < \mu_2$. Americans are taller than English men]

3) The average hourly wage of a sample of 150 workers in plant A was Rs 2.56 with a S.D of Rs 1.08. The average wage of a sample of 200 workers in plant B was Rs 2.87 with a S.D of Rs 1.28. Can an applicant safely assume that the hourly wage paid by plant B are higher than those paid by plant A?

Soln:

Given: plant A: $n_1 = 150$, $\bar{x}_1 = 2.56$, $s_1 = 1.08$

plant B: $n_2 = 200$, $\bar{x}_2 = 2.87$, $s_2 = 1.28$

Step 1: Formulating H_0 and H_1

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$ (one-tailed test)

Step 2: LOS, $\alpha = 5\% = 0.05$

Step 3: Test statistic, $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$

$$= \frac{2.56 - 2.87}{\sqrt{\dfrac{(1.08)^2}{150} + \dfrac{(1.28)^2}{200}}}$$

$$= -2.453$$

$$|z| = |-2.453|$$

$$= 2.453$$

Step 4: Critical value, at 5% LOS is $z_\alpha = 1.645$

Step 5: Conclusion: $z = 2.453 > 1.645 = z_\alpha$

H_0 is rejected at 5% LOS.

$\therefore$ The hourly wage paid by plant B are higher than those paid by plant A.