STUDENT'S t- TEST :

TEST FOR DIFFERENCE OF MEAN:

Null hypothesis : $H_0 : \mu_1 = \mu_2$

Test statistics, $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

where $s^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$ (or) $s^2 = \dfrac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$

Degree of freedom ; $\nu = n_1 + n_2 - 2$.

1) In a test examination given to two groups of students, the marks obtained were as follows:

Group I : 18 20 36 50 49 36 34 49 41

Group II : 29 28 26 35 30 44 46

Examine whether the significance of difference between the average marks secured by the students of the above two groups.

Soln:

Given : Group I : $n_1 = 9$

Group II : $n_2 = 7$.

Now $\bar{x}_1 = \frac{18+20+36+50+49+36+34+49+41}{9} = 37.$

$\bar{x}_2 = \frac{29+28+26+35+30+44+46}{7} = 34.$

x_1	$x_1-\bar{x}_1$	$(x_1-\bar{x}_1)^2$	x_2	$(x_2-\bar{x}_2)$	$(x_2-\bar{x}_2)^2$
18	-19	361	29	-5	25
20	-17	289	28	-6	36
36	-1	1	26	-8	64
50	13	169	35	1	1
49	12	144	30	-4	16
36	-1	1	44	10	100
34	-3	9	46	12	144
49	12	144			
41	4	16			

$\Sigma (x_1-\bar{x}_1)^2 = 1134$ $\qquad$ $\Sigma (x_2-\bar{x}_2)^2 = 386.$

Now $S^2 = \frac{\Sigma (x_1-\bar{x}_1)^2 + \Sigma (x_2-\bar{x}_2)^2}{n_1+n_2-2}$

$= \frac{1134+386}{9+7-2} = 108.57$

$\Rightarrow S = 10.42$

Step 1: Formulating H_0 and H_1

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ (two tailed test)

Step 2: LOS at $\alpha = 5\%$.

Step 3: Test statistic, $t = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$= \dfrac{37 - 34}{10.42\sqrt{\frac{1}{9} + \frac{1}{7}}}$

$= 0.5713$

Step 4: t_{tab} for degrees of freedom, $\nu = n_1 + n_2 - 2$

$= 9 + 7 - 2$

$\nu = 14$

(ii) $t_{tab} = (t_\alpha) = 2.145$

Step 5: Conclusion: $t = 0.5713 < 2.145 = t_\alpha$.

$\therefore$ H_0 is accepted at 5% LOS.

$\therefore$ there is no significant difference in the avg. marks of the two groups of students.

2) A samples of two types of electric bulbs were tested for length of life and the following data were obtained.

Samples	size	Mean	SD
I	8	1134	35
II	7	1024	40

: Test at 5%.

Soln: Given:

Sample I : $n_1 = 8$, $\bar{x}_1 = 1134$, $s_1 = 35$

Sample II : $n_2 = 7$, $\bar{x}_2 = 1024$, $s_2 = 40$.

step 1: Formulating H_0 and H_1.

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (two tailed test)

step 2: LOS at $\alpha = 5\%$.

step 3: Test statistic, $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Now $s^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

$$= \frac{8(35)^2 + 7(40)^2}{8+7-2}$$

$$= 1615.38$$

$$S = 40.19$$

$$\therefore \quad t = \frac{1134 - 1024}{40.19\sqrt{\frac{1}{8} + \frac{1}{7}}}$$

$$= \frac{110}{20.8} = 5.288$$

step 4: t_{tab} for degrees of freedom, $\nu = n_1 + n_2 - 2$

$= 8 + 7 - 2$

$= 13$

(ii) $t_{tab} : t_\alpha = 2.160$.

step 5: Conclusion: $t = 5.288 > 2.160 = t_\alpha$.

$\therefore$ H_0 is rejected at 5%.