CHI - SQUARE TEST :

$$\chi^2 = \frac{\Sigma [O_i - E_i]^2}{E_i}$$

where $O_i \rightarrow$ Observed frequency

$E_i \rightarrow$ Experimental frequency or Expected frequency $= \frac{\Sigma O_i}{n}$

degrees of freedom, $\nu = n - 1$

1) The table below gives the number of aircraft accidents that occured during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Days	Mon	Tues	Wed	Thurs	Fri	Sat
No. of accidents :	14	18	12	11	15	14

Soln:

given, Total no. of accidents = 84

No. of days = 6

$\therefore$ Expected frequencies of the accidents $E_i = \frac{84}{6}$

$= 14$

O_i	E_i	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
14	14	0	$0/14 : 0$
18	14	16	$16/14 : 1.14$
12	14	4	$4/14 : 0.285$
11	14	9	$9/14 : 0.642$
15	14	1	$1/14 : 0.071$
14	14	0	$0/14 : 0$

$$\sum \frac{(O_i-E_i)^2}{E_i} = 2.14285$$

Step 1: Formulate H_0 & H_1:

H_0: The accidents are uniformly distributed.

H_1: The accidents are not uniformly distributed.

Step 2: LOS at $\alpha = 5\%$.

Step 3: Test statistic, $\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = 2.1428$

Step 4: Degrees of freedom, $\nu = n-1$

$= 6-1$

$= 5$

Tab value is $11.07 = \chi^2_\alpha$

Step 5: Conclusion:

$\chi^2 = 2.1428 < 11.07 = \chi^2_\alpha$

$\therefore$ H_0 is accepted at 5% Los. (ie) The accident are uniformly distributed.

2) A die was thrown 498 times. Denoting x to be the number appearing on the top face of it, the observed frequency of x is given below:

x:	1	2	3	4	5	6
f:	69	78	85	82	86	98

what opinion you would form for the accuracy of the die?

Soln: Given, Expected frequency, $E_i = \frac{\text{Total frequency}}{6}$

$= \frac{498}{6} = 83$

Class	O_i	E_i	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
1	69	83	196	2.3614
2	78	83	25	0.3012
3	85	83	4	0.0481
4	82	83	1	0.0120
5	86	83	9	0.1084
6	98	83	225	2.7108

$$\frac{\Sigma (O_i - E_i)^2}{E_i} = 5.5419$$

step 1: Formulate H_0 & H_1:

H_0: A die is unbiased

H_1: A die is not unbiased ie) biased

step 2: LOS at $\alpha = 5\%$.

step 3: Test Statistic, $\chi^2 = \frac{\Sigma (O_i - E_i)^2}{E_i} = 5.542$.

Step 4 : Degrees of freedom, $\nu = n - 1$
$= 6 - 1$
$= 5$
$\therefore \chi^2_\alpha = 11.07$.

Step 5 : Conclusion ;
$\chi^2 = 5.542 < 11.07 = \chi^2_\alpha$
$\therefore$ H_0 is accepted at 5% LOS @ A die is unbiased.

3) The number of automobile accident per week in a certain community as followed 12, 8, 20, 2, 14, 10, 15, 6, 9, 4 are the frequency in agreement with a belief that accident where the same during is 10 week.

Soln: $E_i = \frac{100}{10} = 10$; $\chi^2 = 26.6$; Degrees of freedom : $10 - 1 = 9$
$\chi^2 > \chi^2_\alpha$ at 5% LOS, H_0 is rejected @ the accident condition where not same during 10 week period.

properties:

i) The mean of χ^2 dist. is equal to the no. of degrees of freedom
ii) The variance of χ^2 dist. is twice the degrees of freedom
iii) If χ^2 is a chi-square variate with ν degrees of freedom, then $\chi^2/2$ is a gamma variate with parameter $\nu/2$.
iv) standard χ^2 variate tends to standard normal variate as $n \to \infty$.

Applications:

i) To test if the hypothetical value of the population variance is $\sigma^2 = \sigma_0^2$
ii) To test the goodness of fit.
iii) To test the independence of attributes.
iv) To test the homogeniety of indep. estimates of the population variance.

Degrees of freedom: No. of values in a set which may be assigned arbitrarily.