



DEPARTMENT OF MATHEMATICS

UNIT - II DESIGN OF EXPERIMENTS

LATIN SQUARE :

1) An agriculturist wants to test the effects of four different fertilizers A, B, C and D on the yield of paddy. In order to eliminate sources of error due to variability in self-fertility he used the fertilizers in a Latin square arrangement as given below where the numbers indicate yields in quintals per cent area. Perform an analysis of variance to decide whether there is a difference between the fertilizers at 5% level of significance.

A1218	D2021	C1623	B1011
D1822	A1920	B1110	C1419
B1215	C1521	D1925	A1317
C1622	B1112	A1515	D2024

Soln: Let origin = $n_{ij} - 18$ avs (min, max)

	n_1	n_2	n_3	n_4	total	n_1^2	n_2^2	n_3^2	n_4^2
y_1	0	3	5	-7	1	0	9	25	49
y_2	4	10	-8	1	-1	16	100	64	1
y_3	-3	3	7	-1	6	9	9	49	1
y_4	4	-6	-3	6	1	16	36	9	36
total	5	2	1	-1	7	41	58	147	87
	$\sum n_1$	$\sum n_2$	$\sum n_3$	$\sum n_4$		$\sum n_1^2$	$\sum n_2^2$	$\sum n_3^2$	$\sum n_4^2$



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Step 1: Formulate H_0 & H_1 :

H_0 : There is no difference between the fertilizers.

H_1 : There is difference between the fertilizers.

Step 2: To find T & N :

$$T = \sum n_1 + \sum n_2 + \sum n_3 + \sum n_4$$

$$= 5 + 2 + 1 + 1 = 9$$

$$N = n_1 + n_2 + n_3 + n_4$$

$$= 4 + 4 + 4 + 4 = 16$$

Step 3: To find correction factor, C.F.

$$C.F. = \frac{T^2}{N} = \frac{9^2}{16} = \frac{81}{16} = 5.0625$$

Step 4: To find TSS:

$$TSS = \sum n_1^2 + \sum n_2^2 + \sum n_3^2 + \sum n_4^2 - C.F.$$

$$= 41 + 58 + 147 + 84 - 5.0625$$

$$= 330 - 5.0625 = 324.9375$$

Step 5: To find SSC, SSR, & SST

$$SSC = \frac{(\sum n_1)^2}{n_1} + \frac{(\sum n_2)^2}{n_2} + \frac{(\sum n_3)^2}{n_3} + \frac{(\sum n_4)^2}{n_4} - C.F.$$

$$= \frac{5^2}{4} + \frac{2^2}{4} + \frac{1^2}{4} + \frac{1^2}{4} - 5.0625$$

$$= 4.6875$$



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$$\begin{aligned}SSR &= \frac{(\sum y_1)^2}{n_1} + \frac{(\sum y_2)^2}{n_2} + \frac{(\sum y_3)^2}{n_3} + \frac{(\sum y_4)^2}{n_4} - C.F \\ &= \frac{1^2}{4} + \frac{-1^2}{4} + \frac{6^2}{4} + \frac{1^2}{4} - 3.0625 \\ &= 6.6875\end{aligned}$$

To find SST:

A	0	2	-3	-1	-2 : $\sum z_1$
B	-3	-6	-8	-7	-24 : $\sum z_2$
C	4	3	5	1	13 : $\sum z_3$
D	4	3	7	6	20 : $\sum z_4$

$$\begin{aligned}SST &= \frac{(\sum z_1)^2}{4} + \frac{(\sum z_2)^2}{4} + \frac{(\sum z_3)^2}{4} + \frac{(\sum z_4)^2}{4} - C.F. \\ &= \frac{-2^2}{4} + \frac{-24^2}{4} + \frac{13^2}{4} + \frac{20^2}{4} - C.F. \\ &= 284.25 - 3.0625 = 284.1875\end{aligned}$$

step 6: to find SSE

$$\begin{aligned}SSE &= TSS - SSC - SSR - SST \\ &= 329.94 - 4.6875 - 6.6875 - 284.1875 \\ &= 34.375\end{aligned}$$



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Step 7: Anova table.

Source of variations	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	F-Ratio
Column	SSC: 4.6875	$c-1 = 3$	MSC = $\frac{4.6875}{3}$ = 1.5625	$F_c = \frac{5.7291}{1.5625}$ = 3.6666 $F_{\alpha}(6,3) = 9.94$
Row	SSR: 6.6875	$r-1 = 3$	MSR = $\frac{6.6875}{3}$ = 2.2291	$F_r = \frac{5.7291}{2.2291}$ = 2.5701 $F_{\alpha}(6,3) = 9.94$
Treatment	SST: 284.1875	$T-1 = 3$	MST = $\frac{284.1875}{3}$ = 94.7291	$F_T = \frac{94.7291}{16.5347}$ = 5.7291 $F_{\alpha}(3,6) = 4.76$
Error	SSE: 34.375	$(n-1)(n-2)$ $3 \times 2 = 6$	MSE = $\frac{34.375}{6}$ = 5.7291	

Step 8: Conclusion:

$$F_c = 3.6666 < 9.94 = F_{\alpha}, \text{ } H_0 \text{ is accepted}$$

$$F_r = 2.5701 < 9.94 = F_{\alpha}, \text{ } H_0 \text{ is accepted}$$

$$F_T = 16.5347 > 4.76 = F_{\alpha}, \text{ } H_0 \text{ is rejected}$$

(c) There is difference between the fertilizers.



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2) Analyse the variance in the Latin square of yields (in quintals) of wheat where P, Q, R, S represent the different manures used.

S 222	P 221	R 223	Q 222
Q 224	R 223	P 222	S 225
P 220	Q 219	S 220	R 221
R 222	S 223	Q 221	P 222

Test whether the different manures used have equal significantly different yields:

Soln: $F_c : 1.34$; $F_R : 12.31$, $F_T : 2.12$. & $F_{\alpha} : 4.76$.