Units

Dimensions are measured in terms of units. For example, the dimension of length is measured in terms of length units: the micrometre, millimetre, metre, kilometre, etc.

So that the measurements can always be compared, the units have been defined in terms of physical quantities. For example:

- the metre (m) is defined in terms of the wavelength of light;
- the standard kilogram (kg) is the mass of a standard lump of platinum-iridium;
- the second (s) is the time taken for light of a given wavelength to vibrate a given number of times;
- the degree Celsius (°C) is a one-hundredth part of the temperature interval between the freezing point and the boiling point of water at standard pressure;
- the unit of force, the newton (N), is that force which will give an acceleration of 1 m sec^{-2} to a mass of 1kg;
- the energy unit, the newton metre is called the joule (J), and
- the power unit, 1 J s^{-1}, is called the watt (W).

More complex units arise from equations in which several of these fundamental units are combined to define some new relationship. For example, volume has the dimensions $[L]^3$ and so the units are m^3. Density, mass per unit volume, similarly has the dimensions $[M]/[L]^3$, and the units kg/m^3. A table of such relationships is given in Appendix 1. When dealing with quantities which cannot conveniently be measured in m, kg, s, multiples of these units are used. For example, kilometres, tonnes and hours are useful for large quantities of metres, kilograms and seconds respectively. In general, multiples of 10^3 are preferred such as millimetres (m x 10^{-3}) rather than centimetres (m x 10^{-2}). Time is an exception: its multiples are not decimalized and so although we have micro (10^{-6}) and milli (10^{-3}) seconds, at the other end of the scale we still have minutes (min), hours (h), days (d), etc.

Care must be taken to use appropriate multiplying factors when working with these units. The common secondary units then use the prefixes micro (μ, 10^{-6}), milli (m,10^{-3}), kilo (k, 10^3) and mega (M, 10^6).

Dimensional Consistency

All physical equations must be dimensionally consistent. This means that both sides of the equation must reduce to the same dimensions. For example, if on one side of the equation, the dimensions are $[M]\,[L]/[T]^2$, the other side of the equation must also be $[M]\,[L]/[T]^2$ with the same dimensions to the same powers. Dimensions can be handled algebraically and therefore they can be divided, multiplied, or cancelled. By remembering that an equation must be dimensionally consistent, the dimensions of otherwise unknown quantities can sometimes be calculated.

EXAMPLE 1.1. Dimensions of velocity
In the equation of motion of a particle travelling at a uniform velocity for a time t, the distance travelled is given by $L = vt$. Verify the dimensions of velocity.

Knowing that length has dimensions [L] and time has dimensions [t] we have the dimensional equation:

$[v] = [L]/[t]$

the dimensions of velocity must be $[L][t]^{-1}$

The test of dimensional homogeneity is sometimes useful as an aid to memory. If an equation is written down and on checking is not dimensionally homogeneous, then something has been forgotten.

Unit Consistency and Unit Conversion

Unit consistency implies that the units employed for the dimensions should be chosen from a consistent group, for example in this book we are using the SI (Systeme Internationale de Unites) system of units. This has been internationally accepted as being desirable and necessary for the standardization of physical measurements and although many countries have adopted it, in the USA feet and pounds are very widely used. The other commonly used system is the fps (foot pound second) system and a table of conversion factors is given in Appendix 2.

Very often, quantities are specified or measured in mixed units. For example, if a liquid has been flowing at 1.3 l /min for 18.5 h, all the times have to be put into one only of minutes, hours or seconds before we can calculate the total quantity that has passed. Similarly where tabulated data are only available in non-standard units, conversion tables such as those in Appendix 2 have to be used to convert the units.

EXAMPLE 1.2. Conversion of grams to pounds
Convert 10 grams into pounds.

From Appendix 2, 1lb = 0.4536kg and 1000g = 1kg
so (1lb/ 0.4536kg) = 1 and (1kg/1000g) = 1
therefore 10g =10g x (1lb/0.4536kg) x (1kg/1000g)
= 2.2×10^{-2} lb

$10\text{ g} = 2.2 \times 10^{-2}$ lb

The quantity in brackets in the above example is called a conversion factor. Notice that within the bracket, and before cancelling, the numerator and the denominator are equal. In equations, units can be cancelled in the same way as numbers. Note also that although (1lb/0.4536kg) and (0.4536kg/1lb) are both = 1, the appropriate numerator/denominator must be used for the unwanted units to cancel in the conversion.

EXAMPLE 1.3. Velocity of flow of milk in a pipe.
Milk is flowing through a full pipe whose diameter is known to be 1.8 cm. The only measure available is a tank calibrated in cubic feet, and it is found that it takes 1 h to fill 12.4 ft^3. What is the velocity of flow of the liquid in the pipe in SI units?

Velocity is [L]/[t] and the units in the SI system for velocity are therefore m s^{-1}:

$v = L/t$ where v is the velocity.

Now $V = AL$ where V is the volume of a length of pipe L of cross-sectional area A

i.e. $L = V/A$.

Therefore $v = V/At$
Checking this dimensionally

$$[L][t]^{-1} = [L]^3[L]^{-2}[t]^{-1} = [L][t]^{-1} \text{ which is correct.}$$

Since the required velocity is in m s^{-1}, volume must be in m^3, time in s and area in m^2.
From the volume measurement

$V/t = 12.4 ft^3\ h^{-1}$
From Appendix 2,
$1\ ft^3 = 0.0283\ m^3$
so $1 = (0.0283\ m^3 / 1\ ft^3)$
1 h = 60 x 60 s
so (1 h/3600 s) = 1

Therefore V/t $= 12.4\ ft^3/h \times (0.0283\ m^3/1\ ft^3) \times (1\ h/3600\ s)$
$= 9.75 \times 10^{-5}\ m^3\ s^{-1}$.

Also the area of the pipe A $= \pi D^2/4$
$= \pi(0.018)^2 /4\ m^2$
$= 2.54 \times 10^{-4}\ m^2$
v $= V/t \times 1/A$
$= 9.75 \times 10^{-5}/2.54 \times 10^{-4}$
= <u>0.38 m s^{-1}</u>

EXAMPLE 1.4. Viscosity (μ) conversion from fps to SI units
The viscosity of water at 60°F is given as 7.8×10^{-4} lb ft^{-1} s^{-1}.
Calculate this viscosity in N s m^{-2}.

From Appendix 2,
0.4536 kg = 1 lb
0.3048 m = 1 ft.

$$\text{Therefore } 7.8 \times 10^{-4}\ lb\ ft^{-1}\ s^{-1} = 7.8 \times 10^{-4}\ lb\ ft^{-1}\ s^{-1} \times \frac{0.4536\ kg}{1\ lb} \times \frac{1\ ft}{0.3048 m}$$
$$= 1.16 \times 10^{-3}\ kg\ m^{-1}\ s^{-1}$$

Remembering that one Newton is the force that accelerates unit mass at $1 ms^{-2}$

1 N $= 1\ kg\ m\ s^{-2}$
therefore 1 N s m^{-2} $= 1\ kg\ m^{-1}\ s^{-1}$

<u>Required viscosity = 1.16×10^{-3} N s m^{-2}</u>.

EXAMPLE 1.5. Thermal conductivity of aluminium: conversion from fps to SI units
The thermal conductivity of aluminium is given as 120 Btu ft^{-1} h^{-1} $°F^{-1}$. Calculate this thermal conductivity in J m^{-1} s^{-1} $°C^{-1}$.

From Appendix 2,
1 Btu = 1055 J
0.3048 m = 1 ft
°F = (5/9) °C.

Therefore 120 Btu ft^{-1} h^{-1} $°F^{-1}$

$$= 120 \text{ Btu ft}^{-1} \text{ h}^{-1} \text{ °F}^{-1} \times \frac{1055 \text{ J}}{1 \text{ Btu}} \times \frac{1 \text{ ft}}{0.3048\text{m}} \times \frac{1\text{h}}{3600\text{s}} \times \frac{1\text{°F}}{(5/9)\text{°C}}$$

$$= \underline{208 \text{ J m}^{-1} \text{ s}^{-1} \text{ °C}^{-1}}$$

Alternatively a conversion factor 1Btu ft^{-1} h^{-1} $°F^{-1}$can be calculated:

1Btu $ft^{-1}h^{-1}°F^{-1}$

$$= 1\text{Btu ft}^{-1} \text{ h}^{-1} \text{ °F}^{-1} \times \frac{1055 \text{ J}}{1 \text{ Btu}} \times \frac{1 \text{ ft}}{0.3048 \text{ m}} \times \frac{1\text{h}}{3600\text{s}} \times \frac{1\text{°F}}{(5/9)\text{°C}}$$

$$= 1.73 \text{ J m}^{-1} \text{ s}^{-1} \text{ °C}^{-1}$$

Therefore 120 Btu ft^{-1} h^{-1} $°F^{-1}$

$$= 120 \times 1.73\text{J m}^{-1} \text{ s}^{-1} \text{ °C}^{-1}$$

$$= \underline{208 \text{ J m}^{-1} \text{ s}^{-1} \text{ °C}^{-1}}$$

Because engineering measurements are often made in convenient or conventional units, this question of consistency in equations is very important. Before making calculations always check that the units are the right ones and if not use the necessary conversion factors. The method given above, which can be applied even in very complicated cases, is a safe one if applied systematically.

A loose mode of expression that has arisen, which is sometimes confusing, follows from the use of the word per, or its equivalent the solidus, /. A common example is to give acceleration due to gravity as 9.81 metres per second per second. From this the units of *g* would seem to be m/s/s, that is m s s^{-1} which is incorrect. A better way to write these units would be
$g = 9.81$ m/s^2 which is clearly the same as 9.81 m s^{-2}.

Precision in writing down the units of measurement is a great help in solving problems.

Dimensionless Ratios

It is often easier to visualize quantities if they are expressed in ratio form and ratios have the great advantage of being dimensionless. If a car is said to be going at twice the speed limit, this is a dimensionless ratio, which quickly draws attention to the speed of the car. These dimensionless ratios are often used in process engineering, comparing the unknown with some well-known material or factor.

For example, **specific gravity** is a simple way to express the relative masses or weights of equal volumes of various materials. The specific gravity is defined as the ratio of the weight of a volume of the substance to the weight of an equal volume of water.

SG = weight of a volume of the substance/ weight of an equal volume of water

Dimensionally, $SG = \frac{[F]}{[L]^3} \div \frac{[F]}{[L]^3} = 1$

If the density of water, that is the mass of unit volume of water, is known, then if the specific gravity of some substance is determined, its density can be calculated from the following relationship:

$$\rho = SG\ \rho_w$$

where ρ (rho) is the density of the substance, SG is the specific gravity of the substance and ρ_w is the density of water.

Perhaps the most important attribute of a dimensionless ratio, such as specific gravity, is that it gives an immediate sense of proportion. This sense of proportion is very important to food technologists as they are constantly making approximate mental calculations for which they must be able to maintain correct proportions. For example, if the specific gravity of a solid is known to be greater than 1 then that solid will sink in water. The fact that the specific gravity of iron is 7.88 makes the quantity more easily visualized than the equivalent statement that the density of iron is 7880 kg m^{-3}.

Another advantage of a dimensionless ratio is that it does not depend upon the units of measurement used, provided the units are consistent for each dimension.

Dimensionless ratios are employed frequently in the study of fluid flow and heat flow. They may sometimes appear to be more complicated than specific gravity, but they are in the same way expressing ratios of the unknown to the known material or fact. These dimensionless ratios are then called dimensionless numbers and are often called after a prominent person who was associated with them, for example Reynolds number, Prandtl number, and Nusselt number; these will be explained in the appropriate section.

When evaluating dimensionless ratios, all units must be kept consistent. For this purpose, conversion factors must be used where necessary.

Precision of Measurement

Every measurement necessarily carries a degree of precision, and it is a great advantage if the statement of the result of the measurement shows this precision. The statement of quantity should either itself imply the tolerance, or else the tolerances should be explicitly specified.

For example, a quoted weight of 10.1 kg should mean that the weight lies between 10.05 and 10.149 kg.
Where there is doubt it is better to express the limits explicitly as 10.1 ± 0.05 kg.

The temptation to refine measurements by the use of arithmetic must be resisted. For example, if the surface of a rectangular tank is measured as 4.18 m x 2.22 m and its depth estimated at 3 m, it is obviously unjustified to calculate its volume as 27.8388 m^3 which is what arithmetic or an electronic calculator will give. A more reasonable answer would be 28 m^3. Multiplication of quantities in fact multiplies errors also.

In process engineering, the degree of precision of statements and calculations should always be borne in mind. Every set of data has its least precise member and no amount of mathematics can improve on it. Only better measurement can do this.

A large proportion of practical measurements are accurate only to about 1 part in 100. In some cases factors may well be no more accurate than 1 in 10, and in every calculation proper consideration must be given to the accuracy of the measurements. Electronic calculators and computers may work to eight figures or so, but all figures after the first few