Step 11 :- Final Moment.

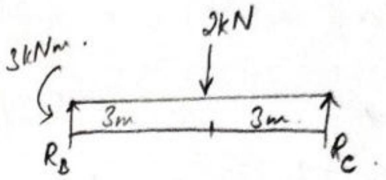
$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -7.47 \\ 3.7 \\ -3.7 \\ 0 \end{bmatrix}$$

Step 12 :- Maximum Bending Moment.

$$M_{AB} = \frac{wl^2}{8} = \frac{3 \times 5^2}{8} = 9.37 \text{ kNm}$$

$$M_{BC} = \frac{wl}{4} = \frac{2 \times 6}{4} = 3 \text{ kNm}$$

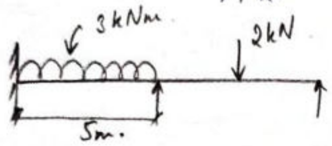
Step 13 :- Shear Force.



$\Sigma V = 0$

$R_A + R_B + R_C = (3 \times 2) + 5$

$= 17$ kN.



Span BC :-

$R_C \times 6 = (2 \times 3) - 3.7$

$R_C = 0.38$ kN.

Span ABC :-

$R_C \times 11 + (R_B \times 5) = (2 \times 8) + (3 \times 5 \times 5/2)$.

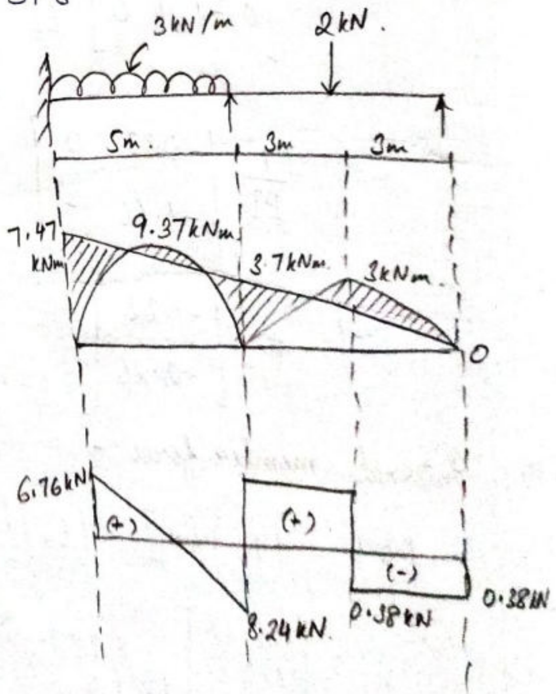
$(11 \times 0.38) + (R_B \times 5) = 16 + 37.5$

$R_B = 9.86$ kN.

$R_A + R_B + R_C = 17$.

$R_A + 9.86 + 0.38 = 17$.

$R_A = 6.76$ kN.



Date :- 5/04/2022

Q3. Analyze the frame as shown in fig. by flexibility matrix method :-

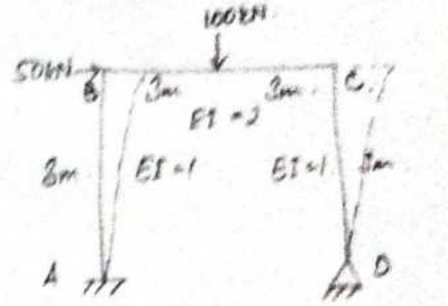
Step 1 :- Static Indeterminacy.

$D_s = R - r$

$= 5 - 3$

$= 2$

D as redundant
As it has two forces.



(General loading condition)

Support D is selected as an redundant.

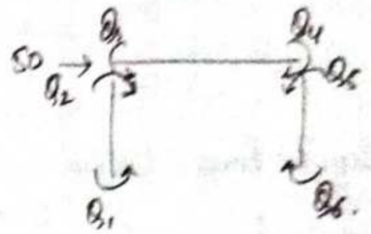
Step 2 :- Fixed End Moments.

$M_{F_{AB}} = M_{F_{BA}} = M_{F_{CD}} = M_{F_{CD}} = 0$

$M_{F_{BC}} = -\frac{wl}{8} = -\frac{100 \times 6}{8} = -75\,kNm$

$M_{F_{CB}} = \frac{wl}{8} = \frac{100 \times 6}{8} = 75\,kNm$

Step 3 :- Internal member force action :-



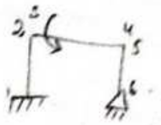
$$Q = \cancel{\begin{Bmatrix} 50 \\ 75 \\ -75 \end{Bmatrix}} \quad \begin{Bmatrix} 0 \\ 0 \\ -75 \\ 75 \\ 0 \\ 0 \end{Bmatrix}$$

Step 4 :- ~~Equivalent~~ Equal & joint moment.

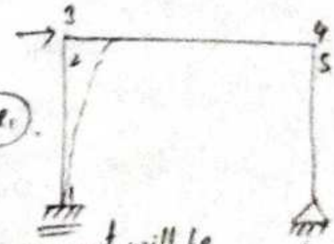
$$R = \begin{Bmatrix} 50 \\ 75 \\ -75 \end{Bmatrix}$$

Step 5 :- Force transformation matrix.

$$\{bR_1\} = \begin{Bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$



$$\{bR_2\} = \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$



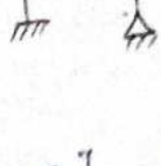
(bR_1).

moment will be created at fix end moment.

pin jointed not created.

Moment - Force × distance

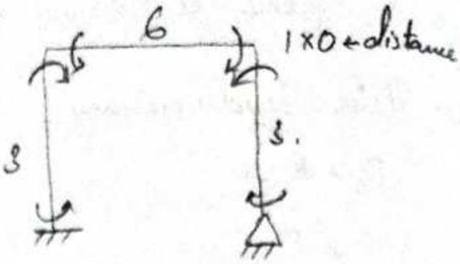
$$\{bR_3\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$



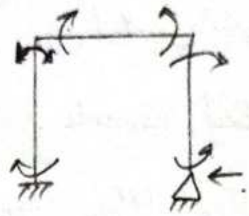
$$\{bR\} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 6 :- Moment transformation matrix :-

$$bx_1 = \begin{Bmatrix} -6 \\ 6 \\ 6 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$



$$bx_2 = \begin{Bmatrix} 0 \\ -3 \\ 3 \\ -3 \\ 3 \\ 0 \end{Bmatrix}.$$



$$bx = \begin{Bmatrix} -6 & 0 \\ 6 & -3 \\ -6 & 3 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{Bmatrix}.$$

Step 7 :- Force Matrix.

$$F_{AB} = \frac{l}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

$$= \frac{3}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

$$= \frac{3}{6\times 1}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}.$$

$$F_{BC} = \frac{l}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

$$= \frac{6}{6\times 2}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}.$$

$$F_{CD} = \frac{3}{6\times 1}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}.$$

$$F = \begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.5 & 0 & 0 \\ 0 & 0 & -0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & -0.5 & 1 \end{bmatrix}.$$

F_x Step 8 :- Flexibility Matrix.

$$F_{xx} = [b_x]^T [F] [b_x].$$

$$= \begin{bmatrix} -6 & 6 & -6 & 0 & 0 & 0 \\ 0 & -3 & 3 & -3 & 3 & 0 \end{bmatrix} \underset{(6\times6)}{\begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.5 & 0 & 0 \\ 0 & 0 & -0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & -0.5 & 1 \end{bmatrix}} \underset{(6\times2)}{\begin{bmatrix} -6 & 0 \\ 6 & -3 \\ -6 & 3 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}}$$

$$= \underset{(2\times6)}{\begin{bmatrix} -6 & 6 & -6 & 0 & 0 & 0 \\ 0 & -3 & 3 & -3 & 3 & 0 \end{bmatrix}} \begin{bmatrix} -6-3 & 4.5 \\ 3+6 & -3 \\ -6 & 3+1.5 \\ \cancel{0}3 & -3-\cancel{1.5} \\ 0 & \cancel{1.5}\,3 \\ 0 & -1.5 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 6 & -6 & 0 & 0 & 0 \\ 0 & -3 & 3 & -3 & 3 & 0 \end{bmatrix} \cancel{\begin{bmatrix} -9 & 1.5 \\ 9 & -3 \\ -6 & 4.5 \\ 0 & -4.5 \\ 0 & -1.5 \\ 0 & \end{bmatrix}} \begin{bmatrix} -9 & 1.5 \\ 9 & -3 \\ -6 & 4.5 \\ 3 & -4.5 \\ 0 & 3 \\ 0 & -1.5 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{+54+54+84}\ 36 & -9-18-27 \\ -27+18 & 9+13.5+9\cancel{3.5+9} \end{bmatrix}.$$

$$\longrightarrow \quad = \begin{bmatrix} 54+54+36 & -9-18-27 \\ -27-18-9 & 9+13.5+9+13.5 \end{bmatrix}$$

$$= \begin{bmatrix} 144 & -54 \\ -54 & 45 \end{bmatrix}.$$

Step 9 :-

$$[F_{xx}] = \begin{bmatrix} 144 & -54 \\ -54 & 45 \end{bmatrix}.$$

$$[A^{-1}] = \frac{1}{|A|}(\text{adj } A).$$

$$[F_{xx}]^{-1} = \underbrace{\frac{1}{3564}}_{=0.00028} \begin{bmatrix} 45 & 54 \\ 54 & 144 \end{bmatrix} = \begin{bmatrix} 0.012 & 0.015 \\ 0.015 & 0.040 \end{bmatrix}.$$

Step 9 :- Flexibility matrix).

$$[F_{xR}] = (b_x)^T (F)(b_R).$$

$$= \begin{bmatrix} -6 & 6 & -6 & 0 & 0 & 0 \\ 0 & -3 & 3 & -3 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.5 & 0 & 0 \\ 0 & 0 & -0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & -0.5 & 1 \end{bmatrix}_{(6\times6)}$$

$$= \begin{bmatrix} -6 & 6 & -6 & 0 & 0 & 0 \\ 0 & -3 & 3 & -3 & 3 & 0 \end{bmatrix}_{(2\times6)} \begin{bmatrix} 3 & 0 & 0 \\ -1.5 & 0 & 0 \\ 0 & 1 & -0.5 \\ 0 & -0.5 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(6\times3)} \qquad \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(6\ldots}$$

$$= \begin{bmatrix} -18 & -18 & 0 \\ 0 & 9+1.5 & -3 \end{bmatrix}.$$

$$= \begin{bmatrix} -18 & -18 & 0 \\ 0 & 10.5 & -3 \end{bmatrix}.$$

$$= \begin{bmatrix} -18-9 & -6 & 3 \\ 13.5 & 3+1.5 & -1.5-3 \end{bmatrix}$$

$$= \begin{bmatrix} -27 & -6 & 3. \\ 4.5. & 45 & -4.5. \end{bmatrix}.$$

Step 10:- Internal member force.

$$[Q_i] = [b_R][R] + [b_x][x] + FEM.$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 50 \\ 75 \\ -75 \end{bmatrix} + \begin{bmatrix} -6 & 0 \\ 6 & -3 \\ -6 & 3 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 10.8 \\ -5.63 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -75 \\ 75 \\ 0 \\ 0 \end{bmatrix}$$

$$[X] = -[F_{XX}]^{-1}[F_{XR}][R]$$

$$= -\begin{bmatrix} 0.012 & 0.015 \\ 0.015 & 0.040 \end{bmatrix} \underset{(2\times3)}{\begin{bmatrix} -27 & -6 & 3 \\ 4.5 & 4.5 & -4.5 \end{bmatrix}} \underset{(3\times1)}{\begin{bmatrix} 50 \\ 75 \\ -75 \end{bmatrix}}$$

$$= -\begin{bmatrix} 0.012 & 0.015 \\ 0.015 & 0.040 \end{bmatrix} \begin{bmatrix} -1350-450-225 \\ 225+337.5+337.5 \end{bmatrix}$$

$$= -\underset{(2\times2)}{\begin{bmatrix} 0.012 & 0.015 \\ 0.015 & 0.040 \end{bmatrix}} \underset{(2\times1)}{\begin{bmatrix} -2025 \\ 900 \end{bmatrix}}$$

$$= -\begin{bmatrix} -24.3+13.5 \\ -30.37+36 \end{bmatrix}$$

$$[X] = \begin{bmatrix} 10.8 \\ -5.63 \end{bmatrix}$$

10. Internal member force :-

$$[Q] = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 50 \\ 75 \\ -75 \end{bmatrix} + \begin{bmatrix} -6 & 0 \\ 6 & -3 \\ -6 & 3 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 10.8 \\ -5.63 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -75 \\ 75 \\ 0 \\ 0 \end{bmatrix}$$

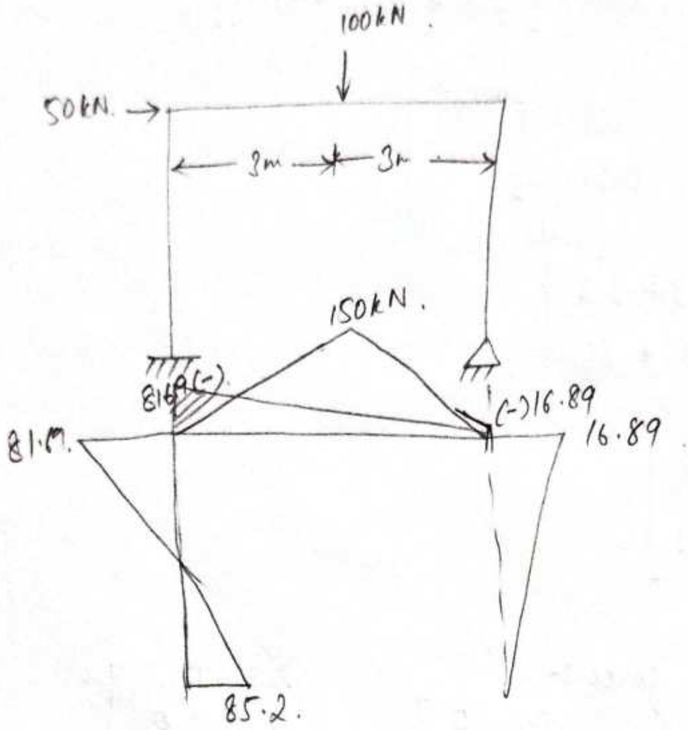
$$= \begin{bmatrix} 150 \\ 0 \\ 75 \\ -75 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -64.8 \\ +64.8+16.89 \\ -64.8-16.89 \\ 16.89 \\ -16.89 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -75 \\ 75 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 150 \\ 0 \\ 75 \\ -75 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -64.8 \\ 81.69 \\ -81.69 \\ +16.89 \\ -6.89 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -75 \\ 75 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 85.2 \\ 81.69 \\ -81.69 \\ 16.89 \\ -16.89 \\ 0 \end{bmatrix}$$

Step 11:- Final Moment.

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} 85.2 \\ 81.69 \\ -81.69 \\ -16.89 \\ 16.89 \\ 0 \end{bmatrix}$$

Step 12:- Maximum Bending Moment.

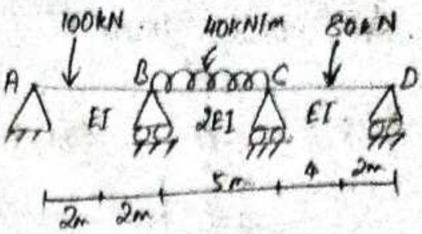


$$\text{Max. B.M.} = \frac{wl}{4} = \frac{100 \times 6}{4}.$$

$$= 150 \text{ kNm}.$$

Date:- 11/04/2021

Q. Analyse the continuous beam as shown in fig. by flexibility matrix method :



(Vertical Loading Condition).

Ans:- Step 1: Static Indeterminacy.

$D_s = R - r$

$= (5-3)\ 4-2$

$= 2$

M_B and M_C are considered to be redundant. $\frac{wab^2}{l^2} \Big/ \frac{wa^2b}{l^2}$

Step 2:- Fixed End Moments.

max B.M. $= \frac{wab}{l}$

$\left[\cancel{M_F} \ M_{FAB} = \frac{wab^2}{l^2} = 100 \times \right]$

$M_{FAB} = -\frac{wl}{8} = -\frac{100 \times 4}{8} = -50$ kNm

$M_{FBA} = \frac{wl}{8} = 50$ kNm.

$M_{FBC} = -\frac{wl^2}{12} = -\frac{40 \times 5^2}{12} = -83.3$ kNm.

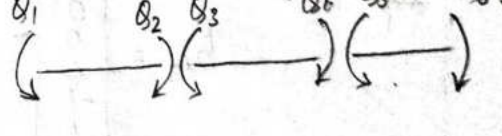
$M_{FCB} = \frac{wl^2}{12} = 83.3$ kNm.

$M_{FCD} = -\frac{wab^2}{l^2} = \frac{80 \times 4 \times 2^2}{6^2} = -35.56$ kNm.

$M_{FDC} = \frac{wa^2b}{l^2} = \frac{80 \times 4^2 \times 2}{6^2} = 71.1$ kNm.

Step 3:- Internal ~~mem~~ member force action.

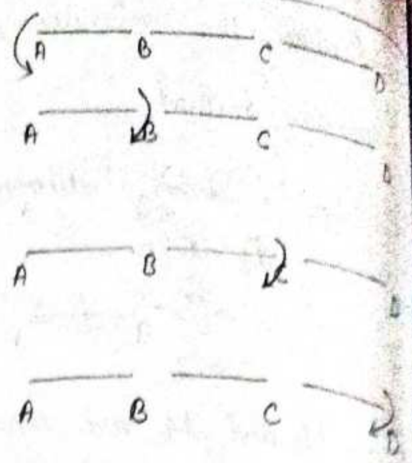
$$Q = \begin{bmatrix} -50 \\ 50 \\ -83.3 \\ 83.3 \\ -35.56 \\ 71.1 \end{bmatrix}.$$



Step 4 :- Equivalent joint moment.

$$R = \begin{bmatrix} 50 \\ 33.3 \\ -47.8 \\ -71.1 \end{bmatrix}$$

Step 5:- Force transformation matrix.



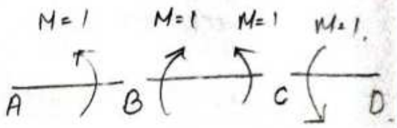
Apply unit force at every points.

$$\{bR_1\} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \{bR_2\} = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$

$$\{bR_3\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix} \qquad \{BR_4\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}.$$

$$\{bR\} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Step 6:- Moment transformation matrix (b_x).



$$b_{x_1} = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -0 \\ 0 \\ 0 \end{bmatrix}.$$

~~Step 7~~ :-

$$b_{x_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \qquad b_x = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Step 7:- Force Matrix.

$$F_{AB} = \frac{l}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \frac{4}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{EI}. \begin{bmatrix} 1.34 & -0.67 \\ -0.67 & 1.34 \end{bmatrix}.$$

$$F_{BC} = \frac{5}{6EI\times 2} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1.66 & -0.83 \\ -0.83 & 1.66 \end{bmatrix} \frac{1}{2EI} \rightarrow \begin{bmatrix} 0.83 & -0.42 \\ -0.42 & 0.83 \end{bmatrix}.$$

$$\begin{bmatrix} = \cancel{(2.76 - 0.69)EI} \\ = 2.07EI. \end{bmatrix}$$

$$F_{CD} = \frac{6}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} \overset{2}{\cancel{1.66}} & \overset{-1}{\cancel{-0.83}} \\ -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI}\begin{bmatrix} 1.34 & -0.67 & 0 & 0 & 0 & 0 \\ -0.67 & 1.34 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overset{0.83}{\cancel{1.66}} & \overset{-0.42}{\cancel{-0.83}} & 0 & 0 \\ 0 & 0 & \overset{-0.42}{\cancel{-0.83}} & \overset{0.83}{\cancel{1.66}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

Step 8 :- Flexibility Matrix .

$$F_{xx} = [b_x]^T [F] [b_x] .$$

$$= \frac{1}{EI}\begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \underset{(6\times 6)}{\begin{bmatrix} 1.34 & -0.67 & 0 & 0 & 0 & 0 \\ -0.67 & 1.34 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.83 & -0.42 & 0 & 0 \\ 0 & 0 & -0.42 & 0.83 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}} \underset{(6\times 2)}{\begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}.$$

$$= \frac{1}{EI}\underset{(2\times 6)}{\begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}} \underset{(6\times 2)}{\begin{bmatrix} 0.67 & 0 \\ -1.34 & 0 \\ 0.83 & 0.42 \\ -0.42 & -0.83 \\ 0 & 2 \\ 0 & -1 \end{bmatrix}}$$

$$= \frac{1}{EI}\begin{bmatrix} 1.34 + 0.83 & 0.42 \\ 0.42 & 0.83 + 2 \end{bmatrix}$$

$$[F_{xx}] = \frac{1}{EI}\begin{bmatrix} 2.17 & 0.42 \\ 0.42 & 2.83 \end{bmatrix} \quad (6.14 - 0.18)$$

~~Step 9.-~~

$$[A] = \frac{1}{|A|}\ (\mathrm{adj}\ A).$$

$$= \frac{EI}{5.96} \begin{bmatrix} 2.83 & -0.42 \\ -0.42 & 2.17 \end{bmatrix}$$

$$= EI \begin{bmatrix} 0.47 & -0.07 \\ -0.07 & 0.36 \end{bmatrix}$$

Step 9:- Internal member force.

$$[S] = [b_R][R] + [b_X][X] + FEM.$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 33.3 \\ -47.8 \\ -71.1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Step 9:- Redundant Force Matrix.

$$[X] = -[F_{XX}]^{-1}[F_{XR}][R].$$

$$[F_{XR}] = [b_X]^T[F][b_R]$$

$$= \begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 1.34 & -0.67 & 0 & 0 & 0 & 0 \\ -0.67 & 1.34 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.83 & -0.42 & 0 & 0 \\ 0 & 0 & -0.42 & 0.83 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}_{(6\times6)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{(6\times4)}.$$

$$= \frac{1}{EI} \begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}_{(2\times4)} \begin{bmatrix} 1.34 & -0.67 & 0 & 0 \\ -0.67 & 1.34 & 0 & 0 \\ 0 & 0 & -0.42 & 0 \\ 0 & 0 & 0.83 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}_{(6\times4)}$$

$$= \frac{1}{EI} \begin{bmatrix} 0.67 & -1.34 & -0.42 & 0 \\ 0 & 0 & -0.83 & -1 \end{bmatrix}.$$

$$[X] = -[F_{XX}]^{-1}[F_{XR}][R].$$

$$\begin{bmatrix} 50 \\ 33.3 \\ -47.8 \\ -71.1 \end{bmatrix}_{(4\times1)}$$

$$= -\frac{\cancel{EI}}{\cancel{EI}}\begin{bmatrix} 0.47 & -0.07 \\ -0.07 & 0.36 \end{bmatrix} \times \frac{1}{\cancel{EI}}\begin{bmatrix} 0.67 & -1.34 & -0.42 & 0 \\ 0 & 0 & -0.83 & -1 \end{bmatrix}_{(2\times4)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{(6\times}$$

$$= 1\begin{bmatrix} 0.47 & -0.07 \\ -0.07 & 0.36 \end{bmatrix}\begin{bmatrix} 33.5 - 44.62 + 20.07 + 0 \\ 39.67 + 71.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.47 & -0.07 \\ -0.07 & 0.36 \end{bmatrix}\cdot\begin{bmatrix} 8.95 \\ 110.77 \end{bmatrix}.$$

$$= \begin{bmatrix} 4.206 - 7.753 \\ -0.6265 + 39.877 \end{bmatrix}$$

16.91

-13.57.

$$= \begin{bmatrix} 3.55 \\ -39.25 \end{bmatrix}.$$

Step 10 :- Internal member force

$$[Q] = [b_R][R] + [b_x][X] + FEM.$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{(6\times4)}\begin{bmatrix} 50 \\ 33.3 \\ -47.8 \\ -71.1 \end{bmatrix}_{(4\times1)} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} +3.55 \\ -39.25 \end{bmatrix} + \begin{bmatrix} -50 \\ 50 \\ -83.3 \\ 83.3 \\ -35.56 \\ 71.1 \end{bmatrix}.$$

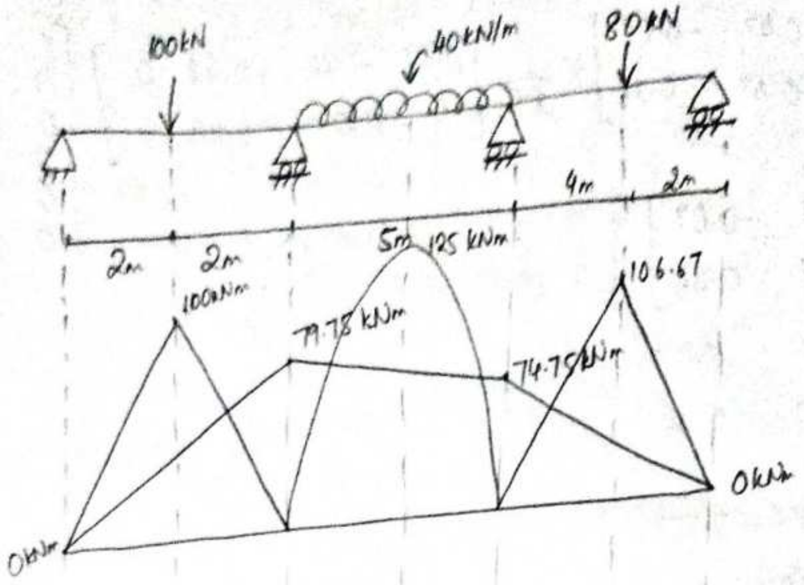
$$= \begin{bmatrix} 50 \\ 33.3 \\ 0 \\ -47.8 \\ 0 \\ -71.1 \end{bmatrix} + \begin{bmatrix} 0 \\ -3.55 \\ 3.55 \\ 39.25 \\ -39.25 \\ 0 \end{bmatrix} + \begin{bmatrix} -50 \\ 50 \\ -83.3 \\ 83.3 \\ -35.56 \\ 71.1 \end{bmatrix} = \begin{bmatrix} 0 \\ 79.75 \\ -79.75 \\ -74.75 \\ -74.81 \\ 0 \end{bmatrix}$$

Step 11:- Final Moment.

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \cancel{\begin{bmatrix} 0 \\ 86.85 \\ -86.85 \\ 3.75 \\ 3.69 \\ 0 \end{bmatrix}} \begin{bmatrix} 0 \\ 79.75 \\ -79.75 \\ 74.75 \\ -74.81 \\ 0 \end{bmatrix}$$

Date:- 13/04/2022

Flexibility Method :-



Bending Moment

$$M_{AB} = \frac{wl}{4} = \frac{100 \times 4}{4} = 100\,kNm.$$

$$M_{BC} = \frac{wl^2}{8} = \frac{40 \times 5^2}{8} = 125\,kNm.$$

$$M_{CD} = \frac{wab}{l} = \frac{80 \times 4 \times 2}{6} = 106.67\,kNm.$$