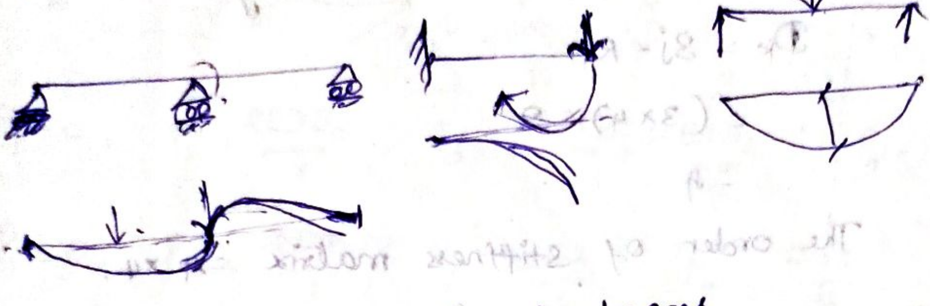
Unit-2 Stiffness Matrix Method

* Suitable for analysing structures by computer applications.
* Used to analyze framed structures.
* Primary unknowns displacements in the form of translation and rotation.



$$K.I = D_K = 3j - R \Rightarrow \text{beams}$$
$$D_K = 2j - R \Rightarrow \text{Truss}$$
$$D_K = 3j - (m+r) \Rightarrow \text{Frames}$$

Stiffness of the Spring:

Load required to produce unit displacement is called Stiffness.

$$[S] = [F]^{-1}$$

$$\text{Flexibility} = \frac{\text{Displacement}}{\text{unit force}}$$

$$\text{Stiffness} = \frac{\text{Force}}{\text{unit displacement}}$$

	Moment	
Support condition	Near end	Far end
Simply supported	$\frac{4EI}{l}$	$\frac{2EI}{l}$

Properties of Stiffness Matrix:

1. It is a square matrix
2. It is a symmetric matrix
3. The diagonal elements are always positive
4. The values of a diagonal elements are high when compared to remaining elements.

1. Determine the stiffness matrix of the beam as shown in figure.

$R = 8$

Solution :

Step 1: Kinematic Indeterminacy :

$D_k = 3j - R$

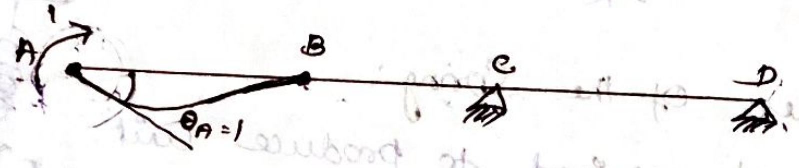
$= (3 \times 4) - 8$

$= 4$

The order of stiffness matrix $= 4 \times 4$.

Step 2 : Stiffness matrix.

For $[k_1]$ apply unit rotation at 1, and restrain 2, 3, 4. $\theta_A = 1$ $\theta_B = \theta_C = \theta_D = 0$.



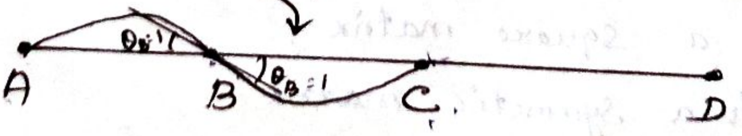
$P_1 = \dfrac{4EI\theta_A}{l_{AB}}$, $P_2 = \dfrac{2EI\theta_A}{l_{BA}}$ $P_3 = P_4 = 0$.

$\theta_A = 1$, $l_{AB} = l_{BC} = l_{CD} = l$

$P_1 = \dfrac{4EI}{l}$, $P_2 = \dfrac{2EI}{l}$, $P_3 = P_4 = 0$.

$$[k_1] = \begin{bmatrix} 4EI/l \\ 2EI/l \\ 0 \\ 0 \end{bmatrix}$$

For k_2 $\theta_B = 1$, $\theta_A = \theta_C = \theta_D = 0$.



$P_1 = \dfrac{2EI\theta_B}{l}$, $P_2 = \dfrac{4EI\theta_B}{l} + \dfrac{4EI\theta_B}{l}$, $P_3 = \dfrac{2EI\theta_B}{l}$, $P_4 = 0$

$P_1 = \dfrac{2EI}{l}$, $P_2 = \dfrac{8EI}{l}$, $P_3 = \dfrac{2EI}{l}$, $P_4 = 0$.

$$[K_2] = \begin{bmatrix} 2EI/l \\ 8EI/l \\ 2EI/l \\ 0 \end{bmatrix}$$

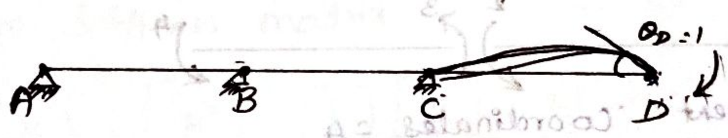
For K_3, $\theta_C = 1$, $\theta_A = \theta_B = \theta_D = 0$.

$P_1 = 0$, $P_2 = \frac{2EI\theta_C}{l}$, $P_3 = \frac{4EI\theta_C}{l} + \frac{4EI\theta_C}{l}$, $P_4 = \frac{2EI\theta_C}{l}$

$P_1 = 0$, $P_2 = \frac{2EI}{l}$, $P_3 = \frac{8EI}{l}$, $P_4 = \frac{2EI}{l}$.

$$[K_3] = \begin{bmatrix} 0 \\ 2EI/l \\ 8EI/l \\ 2EI/l \end{bmatrix}$$

For K_4 $\theta_D = 1$, $\theta_A = \theta_B = \theta_C = 0$



$P_1 = 0$, $P_2 = 0$, $P_3 = \frac{2EI\theta_D}{l}$, $P_4 = \frac{4EI\theta_D}{l}$.

$P_1 = 0$, $P_2 = 0$, $P_3 = \frac{2EI}{l}$, $P_4 = \frac{4EI}{l}$.

$$K_4 = \begin{bmatrix} 0 \\ 0 \\ 2EI/l \\ 4EI/l \end{bmatrix}$$

$$K = \begin{bmatrix} 4EI/l & 2EI/l & 0 & 0 \\ 2EI/l & 8EI/l & 2EI/l & 0 \\ 0 & 2EI/l & 8EI/l & 2EI/l \\ 0 & 0 & 2EI/l & 4EI/l \end{bmatrix}$$

$$k = \frac{EI}{l}\begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 8 & 2 & 0 \\ 0 & 2 & 8 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$