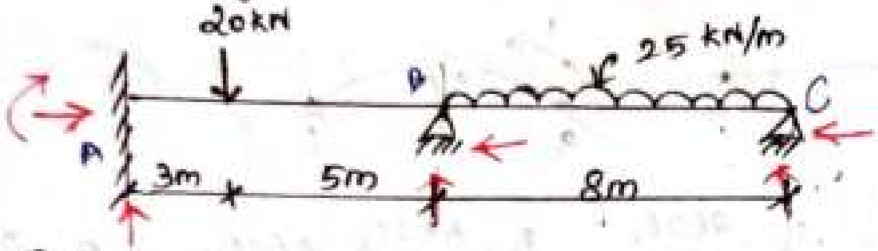
2. A two span continuous beam is fixed at A and hinged over the supports B and C, AB = 8m and BC = 8m. The moment of inertia is constant throughout. It is loaded as shown in fig. Analyse the beam by stiffness matrix method.



$R = 7$

Soln :

Step-1 : kinematic indeterminacy :

$D_K = 3j - R$

$= (3\times3) - 7$

$= 2$

System co-ordinates = 2 (B, C)

Element co-ordinates :

Element coordinates = 4

β matrix = 4×2

Fixed End Moments :

$Mf_{AB} = -\frac{wab^2}{l^2} = -\frac{20\times3\times5^2}{8^2} = -23.43$ kNm

$Mf_{BA} = \frac{wa^2b}{l^2} = \frac{20\times3^2\times5}{8^2} = 14.06$ kNm

$Mf_{BC} = -\frac{wl^2}{12} = -\frac{25\times8^2}{12} = -133.33$ kNm

$Mf_{CB} = \frac{wl^2}{12} = \frac{25\times8^2}{12} = 133.33$ kNm.

$$[P^0] = \begin{bmatrix} -23.43 \\ 14.06 \\ -133.33 \\ 133.33 \end{bmatrix}$$

β matrix :

$$\beta_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\beta_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\beta = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Element Stiffness Matrix :

$$[k_1] = \begin{bmatrix} 4EI/8 & 2EI/8 \\ 2EI/8 & 4EI/8 \end{bmatrix} = EI \begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix}$$

$$k_2 = k_1 \qquad k_n = \begin{bmatrix} 0.5 & 0.25 & 0 & 0 \\ 0.25 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix}$$

System Stiffness matrix : $\frac{2EI}{8}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$[K] = [\beta]^T [k][\beta]$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} EI \begin{bmatrix} 0.5 & 0.25 & 0 & 0 \\ 0.25 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= EI \begin{bmatrix} 0.25 & 0.5 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\frac{4EI}{l}$

$\frac{2EI}{l}$

$$= EI \begin{bmatrix} 1 & 0.25 \\ 0.25 & 0.5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

$$[K]^{-1} = \frac{1}{EI} \begin{bmatrix} 1.143 & -0.571 \\ -0.571 & 2.28 \end{bmatrix}$$

System displacement :

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$[U] = [K]^{-1} \{ f^1 - f^0 \}$$

$$= \frac{EI}{4} \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \qquad f^0 = [\beta]^T [P^0]$$

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$$

$$f^{o} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -23.43 \\ 14.06 \\ -133.33 \\ 133.33 \end{bmatrix}$$

$$f^{o} = \begin{bmatrix} -119.33 \\ 133.33 \end{bmatrix} \quad f^{J} = 0$$

$$U = \frac{1}{EI} \begin{bmatrix} 1.143 & -0.571 \\ -0.571 & 2.28 \end{bmatrix} \begin{bmatrix} 119.33 \\ -133.33 \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 212.52 \\ -372.12 \end{bmatrix}$$

Element displacement (δ):

$$(\delta) = \{B\}\{U\}$$

$$= \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 212.52 \\ -372.12 \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 0 \\ 212.52 \\ 212.52 \\ -372.12 \end{bmatrix}$$

Element forces (p')

$$P' = [K][\delta]$$

$$= EI \begin{bmatrix} 0.5 & 0.25 & 0 & 0 \\ 0.25 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 0 \\ 212.52 \\ 212.52 \\ -372.12 \end{bmatrix}$$

$$= \begin{bmatrix} 53.13 \\ 106.26 \\ 13.23 \\ -132.93 \end{bmatrix}$$

Final forces (p^{f}):

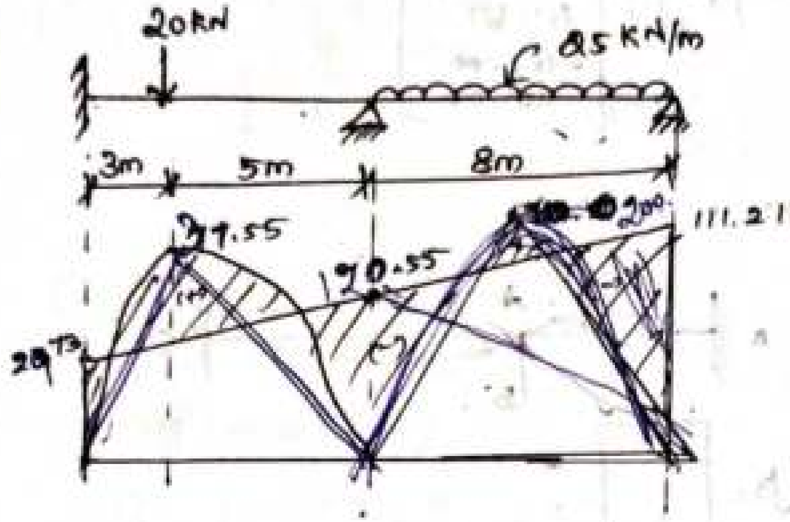
$$P^{f} = [P^{o}] + [P']$$

$$= \begin{bmatrix} -23.43 \\ 14.06 \\ -133.33 \\ 133.33 \end{bmatrix} + \begin{bmatrix} 53.13 \\ 106.26 \\ 13.23 \\ -133 \end{bmatrix} = \begin{bmatrix} 29.7 \\ 120.32 \\ -120.1 \\ 0 \end{bmatrix}$$

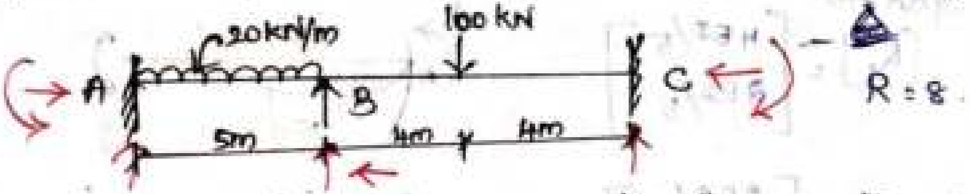
Maximum Bending moments:

$$M_{AB} = \frac{Wab}{l} = \frac{20 \times 3 \times 5}{8} = 37.5 \text{ kNm}$$

$$M_{BC} = \frac{wl^2}{8} = \frac{25 \times 8^2}{8} = 200 \text{ kNm}$$



3. Analyse the continuous beam as shown in figure by stiffness method and draw the bending moment diagram.



Soln: Both ends are fixed, so the center will be a pin jointed only.

Kinematic Indeterminacy:

$$D_k = 3j - R$$
$$= (3\times3) - 8$$
$$= 1$$

System co-ordinate = 1 (B)

Element co-ordinates:

Fixed End moments:

$$MF_{AB} = -\frac{wl^2}{12} = -\frac{20\times5^2}{12} = -41.67 \text{ kNm}.$$

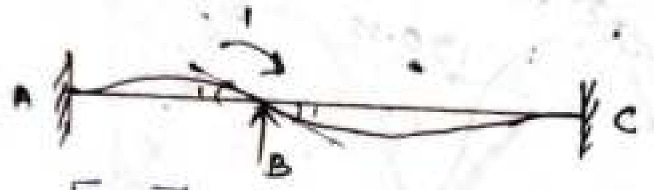
$$MF_{BA} = \frac{wl^2}{12} = \frac{20\times5^2}{12} = 41.67 \text{ kNm}$$

$$MF_{BC} = -\frac{wl}{8} = \frac{100\times 8}{8} = -100\ \text{kNm}.$$

$$MF_{CB} = \frac{wl}{8} = \frac{100\times 8}{8} = 100\ \text{kNm}$$

$$[P^0] = \begin{bmatrix} -41.67 \\ 41.67 \\ -100.00 \\ 100.00 \end{bmatrix}.$$

β matrix :

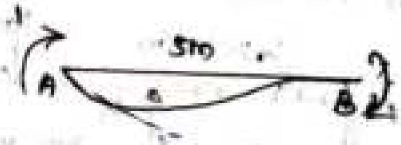


$$\beta = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Element stiffness matrix :

Span AB :

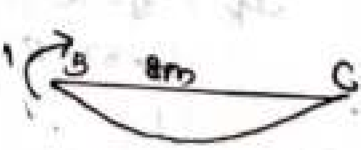
$$[k_1] = \begin{bmatrix} 4EI/5 \\ 2EI/5 \end{bmatrix}$$



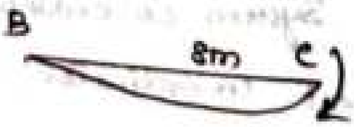
$$k_2 = \begin{bmatrix} 2EI/5 \\ 4EI/5 \end{bmatrix}$$

Span BC :

$$k_3 = \begin{bmatrix} 4EI/8 \\ 2EI/8 \end{bmatrix}$$



$$k_4 = \begin{bmatrix} 2EI/8 \\ 4EI/8 \end{bmatrix}$$



$$[K] = \begin{bmatrix} 0.8 & 0.4 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0.50 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix}$$

System Stiffness matrix:

$$[K] = [B]^T [k] [B]$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} EI \begin{bmatrix} 0.8 & 0.4 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= EI \begin{bmatrix} 0.4 & 0.8 & 0.5 & 0.25 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= 1.3 EI$$

$$[K]^{-1} = \frac{1}{1.3 EI}$$

System displacement $[U]$:

$$[U] = [K]^{-1} \{F^f - F^0\}$$

$[F^f]$ forces applied at system co-ordinates $= 0$.

[∴ there is no external force at the system coordinates]

F^0 = Fixed coordinate forces $= [B]^T [P^0]$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -41.67 \\ 41.67 \\ -100 \\ 100 \end{bmatrix}$$

$$= [-58.33]$$

$$U = \frac{1}{1.3 EI} \left[0 - (-58.33) \right]$$

$$= \frac{58.33}{1.3 EI} = \frac{44.86}{EI}$$

Element displacement (δ):

$$\delta = [B][U]$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \frac{44.86}{EI} = \frac{1}{EI} \begin{bmatrix} 0 \\ 44.86 \\ 44.86 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 0 \\ 44.86 \\ 44.86 \\ 0 \end{bmatrix}$$

Element forces [p'] :

$$[P'] = [k][\delta]$$

$$= EI \begin{bmatrix} 0.8 & 0.4 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 0 \\ 44.86 \\ 44.86 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 17.94 \\ 35.88 \\ 22.43 \\ 11.215 \end{bmatrix}$$

Final forces (pf) :

$$pf = [P^0] + [P']$$

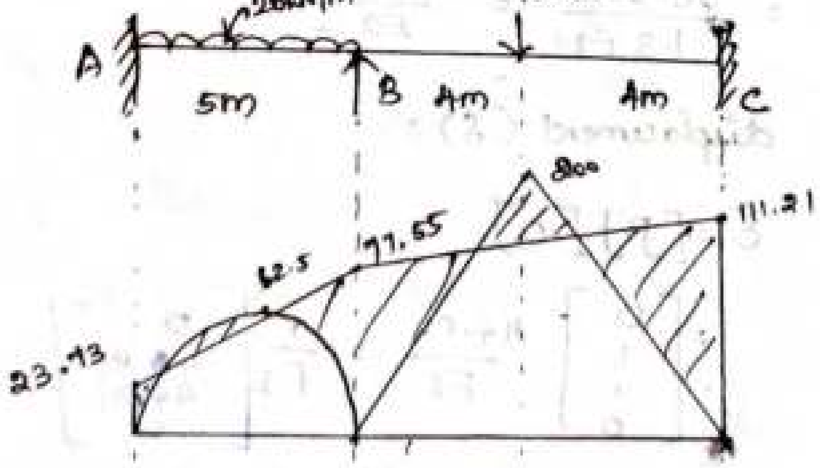
$$= \begin{bmatrix} -41.67 \\ 41.67 \\ -100 \\ 100 \end{bmatrix} + \begin{bmatrix} 17.94 \\ 35.88 \\ 22.43 \\ 11.215 \end{bmatrix}$$

$$= \begin{bmatrix} -23.73 \\ 77.55 \\ -77.55 \\ 111.21 \end{bmatrix}$$

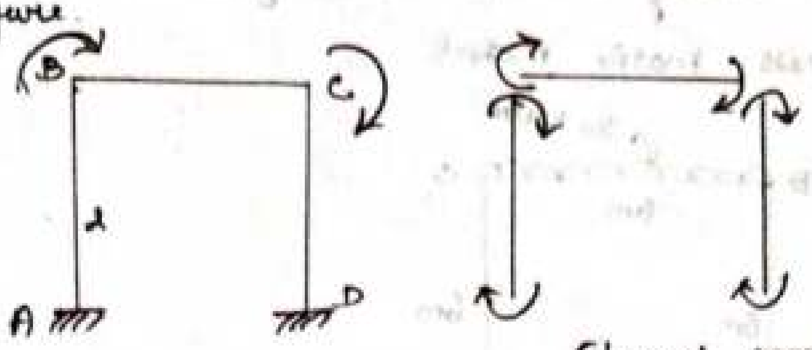
Maximum Bending Moment :

$$M_{AB} = \frac{wl^2}{8} = \frac{20 \times 5^2}{8} = 62.5 \text{ kNm}$$

$$M_{BC} = \frac{wl}{4} = \frac{100 \times 8}{4} = 200 \text{ kNm}$$



4. Generate the β matrix for the portal frame as shown in figure



Element coordinate

Soln :

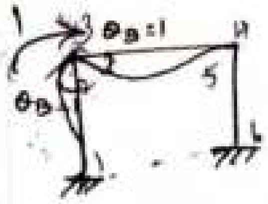
Kinematic indeterminacy :

$$D_K = 3j - (m+r)$$

Here there is 2 system coordinates and 6 element coordinates.

Apply unit rotation at first system coordinate

$$\beta = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

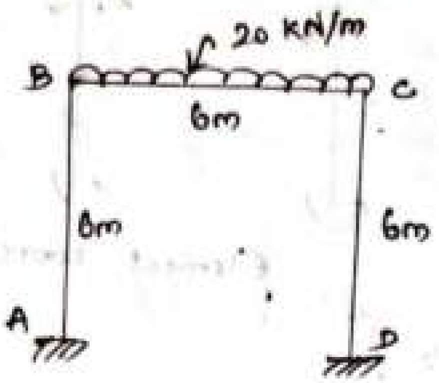


Apply unit rotation at Second system coordinate.

$$\beta = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\beta = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

5. Find Bending moment of the given frame by Stiffness matrix method.

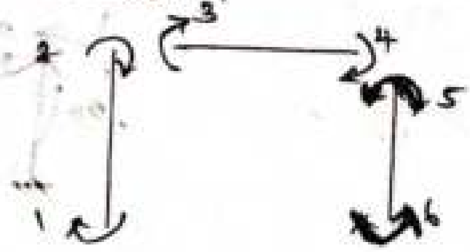


Soln:

Unknown displacements $= \theta_B, \theta_C$.

System coordinates $= 2$ (B and C)

Element coordinates:



Fixed end moments:

$$MF_{AB} = MF_{BA} = 0$$

$$MF_{BC} = \frac{-Wl^2}{12} = \frac{-20 \times 6^2}{12}$$

$$= -60 \text{ kNm}$$

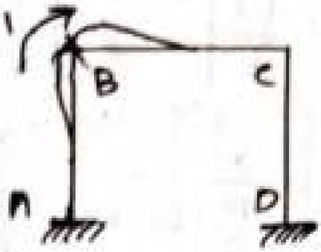
$$MF_{CB} = \frac{wL^2}{12} = \frac{20 \times 6^2}{12} = 60 \text{kNm}$$

$$MF_{CD} = MF_{DC} = 0$$

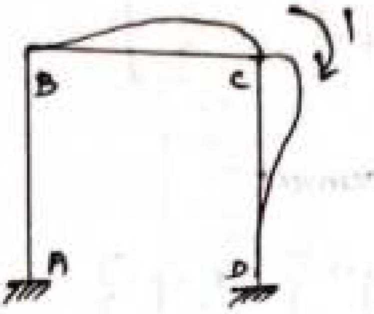
$$P^0 = \begin{bmatrix} 0 \\ 0 \\ -60 \\ 60 \\ 0 \\ 0 \end{bmatrix}$$

β Matrix :

$$\beta_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



$$\beta_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$



$$\beta = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Element ~~Syst~~ Stiffness Matrix :

$$K_1 = \frac{EI}{6}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

$$K_2 = \frac{EI}{6}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \qquad K_3 = \frac{EI}{6}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

$$k = \frac{EI}{6}\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix}$$

System Stiffness Matrix :

$$K = [\beta]^T [k][\beta]$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \frac{EI}{6}\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$K = \frac{EI}{6}\begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix}$$

$$K^{-1} = \frac{1}{\frac{EI}{6}(64-4)}\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix}$$

$$= \frac{1}{10EI}\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix}$$

System displacement:

$$U = [K]^{-1}\left\{ F^f - f^\circ \right\}$$

$$f^\circ = [B]^T[P^\circ]$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -60 \\ 60 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -60 \\ 60 \end{bmatrix}$$

$$U = \frac{1}{10EI}\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix}\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -60 \\ 60 \end{bmatrix}\right\}$$

$$= \frac{1}{EI}\begin{bmatrix} 60 \\ -60 \end{bmatrix}$$

Element displacement:

$$\delta = [B][U]$$

$$= \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}_{(6\times 2)} \frac{1}{EI}\begin{bmatrix} 60 \\ -60 \end{bmatrix}_{(2\times 1)}$$

$$= \frac{1}{EI}\begin{bmatrix} 0 \\ 60 \\ 60 \\ -60 \\ -60 \\ 0 \end{bmatrix}_{(6\times 1)}$$

Element force (p')

$$P = [k][\delta]$$

$$= \frac{EI}{6}\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix}_{(6\times6)} \begin{bmatrix} 0 \\ 60 \\ 60 \\ -60 \\ -60 \\ 0 \end{bmatrix}_{(6\times1)} \frac{1}{EI}$$

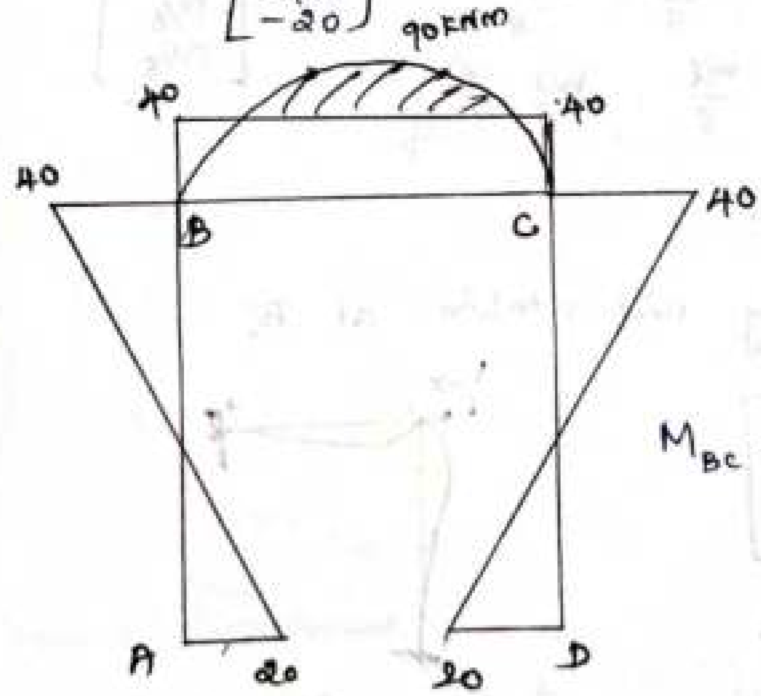
$$= \frac{1}{6}\begin{bmatrix} 120 \\ 240 \\ 120 \\ -120 \\ -240 \\ -120 \end{bmatrix} = \begin{bmatrix} 20 \\ 40 \\ 20 \\ -20 \\ -40 \\ 20 \end{bmatrix}$$

Final force (p^f):

$$p^f = p' + p^\circ$$

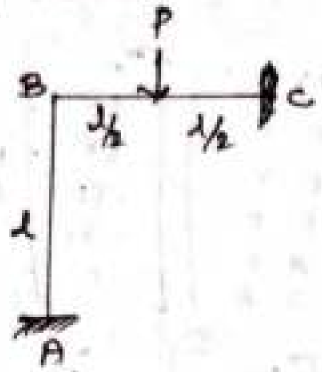
$$= \begin{bmatrix} 20 \\ 40 \\ 20 \\ -20 \\ -40 \\ -20 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -60 \\ 60 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 20 \\ 40 \\ -40 \\ 40 \\ -40 \\ -20 \end{bmatrix} \text{ kNm.}$$



$$M_{BC} = \frac{Wl}{8} = \frac{20\times6}{8} = 90 \text{ kNm.}$$

6. Analyse the frame as shown in figure by Stiffness method



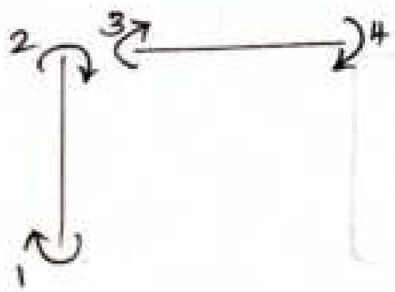
Soln:

System coordinates:

Unknown displacements $= \theta_B$

system coordinate $= 1$ (B)

Element Coordinates:



Fixed end Moments:

$MF_{AB} = MF_{BA} = 0$

$MF_{BC} = -\frac{wl}{8} = -\frac{Pl}{8}$

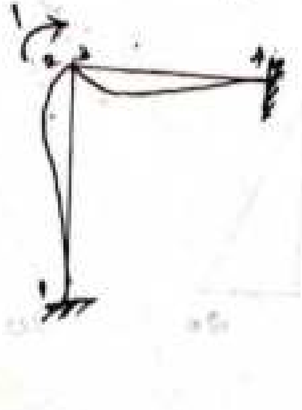
$MF_{CB} = \frac{wl}{8} = \frac{wl}{8} = \frac{Pl}{8}$

$$P^{\circ} = \begin{bmatrix} 0 \\ 0 \\ -Pl/8 \\ Pl/8 \end{bmatrix}$$

β matrix:

Applying unit rotation at B,

$$\beta = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$



Element stiffness matrix :

$$K_1 = \frac{EI}{l}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \quad K_2 = \frac{EI}{l}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

$$k = \frac{EI}{l}\begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

System Stiffness matrix :

$$K = [\beta]^T [k] [\beta]$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} \frac{EI}{l}\begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \frac{8EI}{l}$$

$$K^{-1} = \frac{l}{8EI}$$

System displacement :

$$U = [K]^{-1}\{f^f - f^0\}$$

$$f^0 = [\beta]^T [\phi]$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -Pl/8 \\ Pl/8 \end{bmatrix}$$

$$f^0 = -Pl/8$$

$$f^f - f^0 = 0 - (-Pl/8) = Pl/8$$

$$U = l/_{8EI}\left[Pl/8\right]$$

$$= Pl^2/64EI$$

Element displacement :

$$\delta = [\beta][u]$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} Pl^2/64EI$$

Element ~~displacement~~ force :

$$\phi = [k][\delta] = \frac{EI}{l}\begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}\frac{Pl^2}{64EI}$$

$$P' = \frac{Pl}{64}\begin{bmatrix}2\\4\\4\\2\end{bmatrix}$$

Final forces:

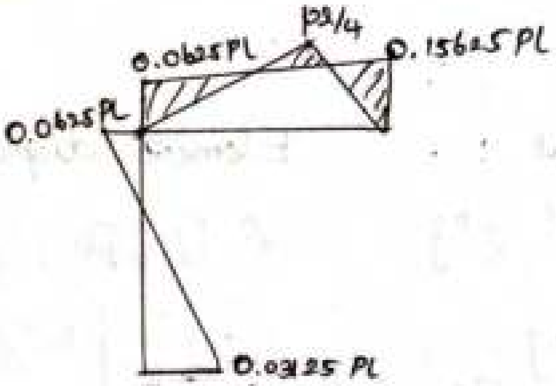
$$P^f = P^0 + P'$$

$$= \begin{bmatrix}0\\0\\-Pl/8\\Pl/8\end{bmatrix} + \frac{Pl}{64}\begin{bmatrix}2\\4\\4\\2\end{bmatrix}$$

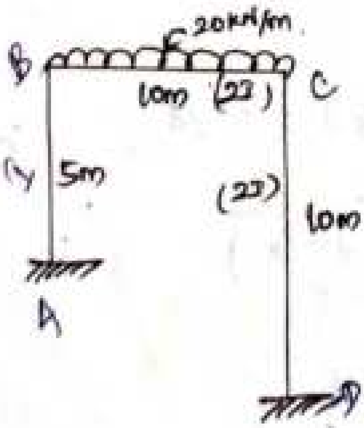
$$= Pl\begin{bmatrix}0.031\\0.0625\\-0.0625\\0.15625\end{bmatrix}$$

Maximum Bending moment:

$$M_{BC} = \frac{Wl}{4} = \frac{Pl}{4} \cdot 0.25Pl$$

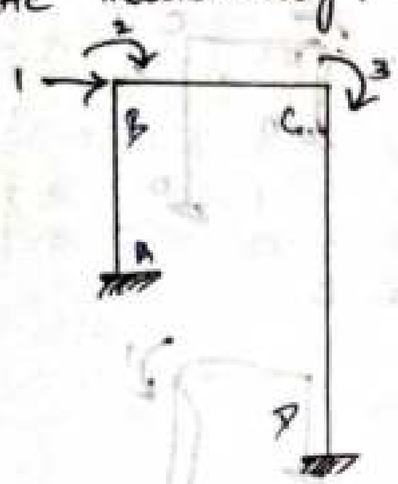


7. Analyze the frame as shown in figure by Stiffness matrix method.



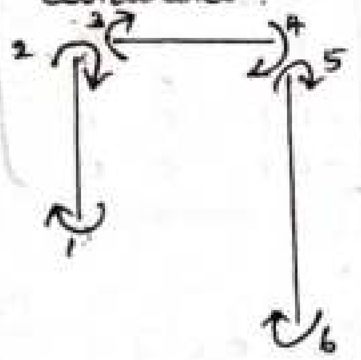
Soln :

i) Kinematic indeterminacy :



This structure is Kinematically indeterminate to 3 degree $(\theta_B, \delta_B, \theta_C)$

Element coordinates :



Element coordinates = 6.

Fixed end moments :

$$MF_{AB} = MF_{BA} = MF_{CD} = MF_{DC} = 0.$$

$$MF_{BC} = -\frac{wl^2}{12} = -\frac{20\times 10^2}{12} = -166.67\ \text{kNm}$$

$$MF_{CB} = \frac{wl^2}{12} = \frac{20\times 10^2}{12} = 166.67\ \text{kNm}.$$

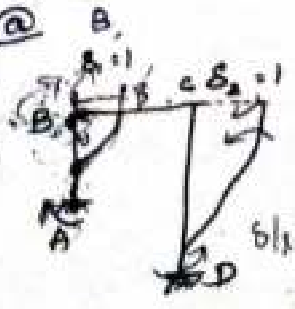
$$P^\circ = \begin{bmatrix} 0 \\ 0 \\ -166.67 \\ 166.67 \\ 0 \\ 0 \end{bmatrix}$$

β matrix :

Applying unit displacement @ B.

$$\beta_1 = \begin{bmatrix} -1/5 \\ -1/5 \\ 0 \\ 0 \\ -1/10 \\ -1/10 \end{bmatrix}$$

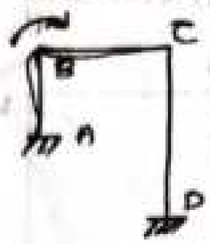
$\delta/l = 1/5$



$\delta/l = 1/10$

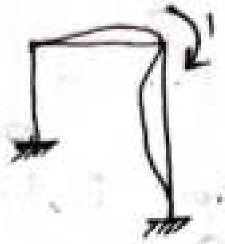
Applying unit rotation @ B,

$$\beta_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



Applying unit rotation @ C

$$\beta_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$



$$\beta_1 = \begin{bmatrix} -0.2 \\ -0.2 \\ 0 \\ 0 \\ -0.1 \\ -0.1 \end{bmatrix} \qquad \beta = \begin{bmatrix} -0.2 & 0 & 0 \\ -0.2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.1 & 0 & 1 \\ -0.1 & 0 & 0 \end{bmatrix}$$

Element Stiffness Matrix :

$$k_1 = \frac{EI}{5}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = EI\begin{bmatrix} 0.8 & 0.4 \\ 0.4 & 0.8 \end{bmatrix}$$

$$k_2 = \frac{E \times 2I}{10}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = EI\begin{bmatrix} 0.8 & 0.4 \\ 0.4 & 0.8 \end{bmatrix}$$

$$k_3 = \frac{E \times 2I}{10}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = EI\begin{bmatrix} 0.8 & 0.4 \\ 0.4 & 0.8 \end{bmatrix}$$

$$[K] = EI\begin{bmatrix} 0.8 & 0.4 & 0 & 0 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \\ 0 & 0 & 0 & 0 & 0.4 & 0.8 \end{bmatrix}$$

System stiffness matrix:

$$K = [\beta]^T[k][\beta]$$

$$= \begin{bmatrix} -0.2 & -0.2 & 0 & 0 & -0.1 & -0.1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}_{(3\times 6)} EI \begin{bmatrix} 0.8 & 0.4 & 0 & 0 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \\ 0 & 0 & 0 & 0 & 0.4 & 0.8 \end{bmatrix}_{(6\times 6)}$$

$$\times \begin{bmatrix} -0.2 & 0 & 0 \\ -0.2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.1 & 0 & 1 \\ -0.1 & 0 & 0 \end{bmatrix}_{(6\times 3)}$$

$$= EI \begin{bmatrix} 0.12 & -0.24 & -0.12 \\ -0.24 & 1.6 & 0.4 \\ -0.12 & 0.4 & 1.6 \end{bmatrix}_{(3\times 3)}$$

System displacement $[U]$:

$$[U] = [K]^{-1}\{f^t - f^0\} \qquad A^{-1} = \frac{1}{|A|}\,\text{Adj}\,A$$

$$[K]^{-1} = \frac{1}{EI}\begin{bmatrix} 12.25 & 1.715 & 0.490 \\ 1.715 & 0.906 & -0.098 \\ 0.490 & -0.098 & 0.686 \end{bmatrix} \qquad f^0 = [\beta][P^0] \qquad f^0 = \begin{bmatrix} 0 \\ -166.67 \\ 166.67 \end{bmatrix}$$

$$[U] = \frac{1}{EI}\begin{bmatrix} 12.25 & 1.715 & 0.490 \\ 1.715 & 0.906 & -0.098 \\ 0.490 & -0.098 & 0.686 \end{bmatrix}\begin{bmatrix} 0 \\ 166.67 \\ -166.67 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 204.17 \\ 167.33 \\ -130.66 \end{bmatrix}$$

Element displacement (δ):

$$\delta = [\beta][U]$$

$$= \begin{bmatrix} -0.2 & 0 & 0 \\ -0.2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.1 & 0 & 1 \\ -0.1 & 0 & 0 \end{bmatrix} \frac{1}{EI}\begin{bmatrix} 204.17 \\ 167.33 \\ -130.66 \end{bmatrix}$$

$$\delta = \frac{1}{EI}\begin{bmatrix} -40.83 \\ 126.496 \\ 167.33 \\ -130.66 \\ -151.07 \\ -20.47 \end{bmatrix}$$

Element Forces:

$$P' = [k][\delta]$$

$$= \frac{EI}{} \begin{bmatrix} 0.8 & 0.4 & 0 & 0 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \\ 0 & 0 & 0 & 0 & 0.4 & 0.8 \end{bmatrix}_{(6\times 6)} \cancel{\frac{1}{EI}} \begin{bmatrix} -40.83 \\ 126.496 \\ 167.33 \\ -130.66 \\ -151.07 \\ -20.47 \end{bmatrix}_{(6\times 1)}$$

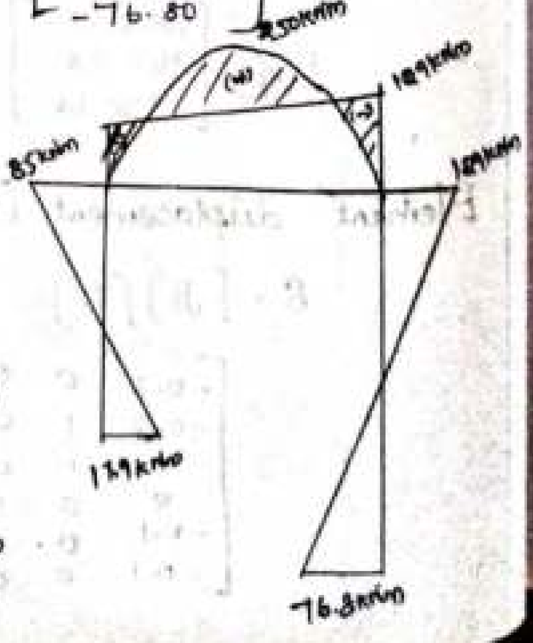
$$= \begin{bmatrix} 17.93 \\ 84.86 \\ 81.60 \\ -37.59 \\ -129.04 \\ -76.80 \end{bmatrix}$$

Final Forces:

$$P^f = P^{\circ} + P'$$

$$= \begin{bmatrix} 0 \\ 0 \\ -166.67 \\ 166.67 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 17.93 \\ 84.86 \\ 81.60 \\ -37.59 \\ -129.04 \\ -76.80 \end{bmatrix}$$

$$= \begin{bmatrix} 17.9 \\ 85 \\ -85 \\ 129 \\ -129 \\ -76.80 \end{bmatrix}$$



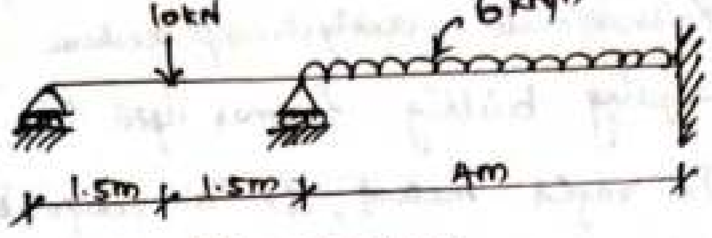
Maximum Bending moment:

$M_{AB} = M_{CD} = 0$

$M_{BC} = \frac{Wl^2}{8} = \frac{20\times10^2}{8} = 250\,kNm$

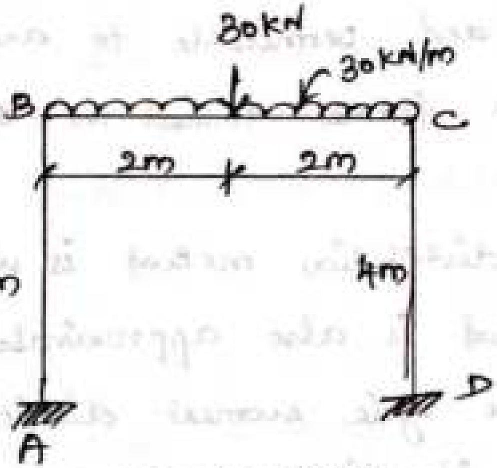
Assignment - 2 Problems :

1. Analyse the continuous beam as shown in figure by stiffness matrix method and sketch the BMD.



EI = constant

2. Analyse the portal frame ABCD shown in figure by stiffness matrix method and also sketch the BMD.



EI = Constant