Boats and Streams

Boats and streams is an application of concepts of speed, time and distance. Speed of river flowing either aides a swimmer (boat), while travelling with the direction of river or it opposes when travelling against the direction of river.

Still water: If the speed of water of a river is zero, then water is considered to be still water.

Stream water: If the water of a river is moving at a certain speed, then it is called as stream water.

Speed of Boat: Speed of boat means speed of boat (swimmer) in still water. In other words, if the speed of a boat (swimmer) is given, then that particular speed is the speed in still water.

Downstream Motion: If the motion of a boat (swimmer) is along the direction of stream, then such motion is called downstream motion.

Upstream Motion: If the motion of a boat (swimmer) is against the direction of stream, then such motion is called upstream motion.

Basic Formulae Related to Boats and Streams

If the speed of a boat in still water is $x\ km/h$ and speed of the stream is $y\ km/h$, then

1. Speed downstream = $(x + y)km/h$
2. Speed upstream = $(x - y)\ km/h$
3. Speed of a boat in still water $(x) = \frac{1}{2}$ (Speed downstream + Speed upstream)
4. Speed of a stream $(y) = \frac{1}{2}$ (Speed downstream – Speed upstream)

Example : 1 A man can row with a speed of 6 km/h in still water. What will be his speed with the stream, if the speed of stream is 2 km/h?

Solution: Given, speed of man in still water = $x = 6\ km/h$ and speed of stream = y=2 km/h

Speed downstream = $x + y = 6 + 2 = 8\ km/h$.

Example : 2 If the speed of a boat in still water is 8 km/h and the rate of stream is 4 km/h, then find upstream speed of the boat.

Solution: Given, speed of a boat = $x = 8\ km/h$

Speed of stream = $y = 4\ km/h$

Speed upstream = $x - y = 8 - 4 = 4\ km/h$

Example : 3 Shantanu can row upstream at 10 km/h and downstream at 18 km/h. Find the man's rate in still water and the rate of the current.

Solution: Speed upstream = 10 km/h and Speed downstream = 18 km/h

According to the formula,

Man's rate in still water = $\frac{1}{2}(Speed\ downstream + Speed\ upstream)$

$$=\frac{1}{2}(18 + 10) = \frac{28}{2} = 14\ km/h$$

Speed of current = $\frac{1}{2}(Speed\ downstream - Speed\ upstream)$

$$=\frac{1}{2}(18 - 10) = \frac{8}{2} = 4\ km/h$$

Example : 4 What time will be taken by a boat to cover a distance of 64 km along the stream, if speed of boat in still water is 12 km/h and speed of stream is 4 km/h?

Solution: Given that, distance = 64 km, speed of boat in still water = $x = 12\ km/h$ and speed of stream = $y = 4\ km/h$

Downstream speed of boat = $x + y = 12 + 4 = 16\ km/h$

Required time = $\frac{\text{Distance}}{\text{Speed(downstream)}} = \frac{64}{16} = 4\ h.$

Example : 5 A boat takes 8 h to row 48 km downstream and 12 h to row the same distance upstream. Find the boat's rate in still water and rate of current.

Solution: Speed downstream = $\frac{Distance}{Time} = \frac{48}{8} = 6\ km/h$

Speed upstream = = $\frac{48}{12} = 4\ km/h$

Now, rate of boat in still water = $\frac{(\text{Speed downstream}+\text{Speed upstream})}{2} = \frac{6+4}{2} = 5\ km/h$

and rate of current = $\frac{(\text{Speed downstream}-\text{Speed upstream})}{2} = \frac{6-4}{2} = 1\ km/h.$

Fast Track Techniques to solve the questions:

Technique :1

If speed of stream is a and a boat (swimmer) takes n times as long to row up as to row down the river, then

Speed of boat (Swimmer) in still water = $\frac{a(n+1)}{(n-1)}$

Note : This formula is applicable for equal distances.

Example : 6 Rajnish can row 12 km/h in still water. It takes him twice as long to row up as to row down the river. Find the rate of stream.

Solution: Here speed of Rajnish in still water = 12 km/h

$n = 2$, Speed of stream (a) =?

According to the formula, Speed in still water = $\frac{a(n+1)}{(n-1)}$

$$12 = \frac{a(2+1)}{(2-1)}$$

$$3a = 12 \Rightarrow a = 4\ km/h$$

Technique : 2

A person can row at a speed of x in still water. If stream is flowing at a speed of y, it takes time T to row to a place and back, then

Distance between two places = $\frac{T(x^2-y^2)}{2x}$

Example : 7 A man can row 12 km/h in still water. When the river is running at 2.4 km/h, it takes him 1 h to row to a place and to come back. How far is the place?

Solution: Here, Speed of man in still water = $x = 12\ km/h$

Speed of river = $= 24\ km/h$; $T = 1\ h$

According to the formula,

Required distance = $\frac{T(x^2-y^2)}{2x} = \frac{1\times[(12)^2-(2.4)^2]}{2\times12} = \frac{138.24}{24} = 5.76\ km$

Technique : 3

A man rows a certain distance downstream in x h and returns the same distance in y h. when the stream flows at the rate of a km/h, then

Speed of the man in still water = $\frac{a(x+y)}{(y-x)}$

Example : 8 Kamal can row a certain distance downstream in 12 h and can return the same distance in 18 h. If the stream flows at the rate of 6 km/h, then find the speed of Kamal in still water.

Solution: Here, $x = 12\ h, y = 18\ h$

Rate of stream (a) = 6 km/h

According to the formula,

Speed of kamal in still water = $\frac{a(x+y)}{(y-x)} = \frac{6\times(12+18)}{(18-12)} = \frac{6\times30}{6} = 30\ km/h$

Note: If in case of technique 3, man's speed in still water is b km/h and we are asked to find the speed of stream, then technique 3 takes the form as

Speed of the stream = $\frac{b(y-x)}{(x+y)}$

Example : 9 If in the above example, the speed of Kamal in still water is 12 km/h, then find the speed of the stream.

Solution: Here, b=12, y=18 and x=12

According to the formula,

Speed of stream = $\frac{b(y-x)}{(x+y)} = \frac{12(18-12)}{18+12} = \frac{12\times6}{30} = \frac{12}{5} = 2.4\ km/h$

Technique : 4

If boat's (swimmer's) speed in still water is a km/h and river Is flowing with a speed of b km/h, then average speed in going to a certain place and coming back to starting point is given by $\frac{(a+b)(a-b)}{a}\ km/h.$

Example : 10 Ramesh rows in still water with a speed of 4.5 km/h to go to a certain place and to come back. Find his average speed for the whole journey, if the river is flowing with a speed of 1.5 km/h.

Solution: Here a= 4.5 km/h , b= 1.5 km/h

Average speed = $\frac{(a+b)(a-b)}{a} = \frac{(4.5+1.5)(4.5-1.5)}{4.5} = \frac{6\times3}{4.5} = \frac{18}{4.5} = 4\ km/h$

Technique : 5

When boat's speed (swimmer's speed) in still water is a km/h and river is flowing with a speed of b km/h and time taken to cover a certain distance upstream is T more than the time taken to cover the same distance downstream, then Distance = $\frac{(a^2-b^2)T}{2b}$

Example : 11 A boat's speed in still water is 10km/h, while river is flowing with a speed of 2 km/h and time taken to cover a certain distance upstream is 4 h more than time taken to cover the same distance downstream. Find the distance.

Solution: Here, a = 10 km/h , b= 2 km/h and T=4 h

According to the formula,

Required distance = $\frac{(a^2-b^2)}{2b} \times T = \frac{(10^2-2^2)}{2\times2} \times 4 = \frac{100-4}{4} \times 4 = 100-4 = 96\ km$

Technique : 6

If a man covers l km distance in $t_1\ h$ along the direction of river and he covers same distance in $t_2\ h$ against the direction of river, then

Speed of man = $\frac{1}{2}\left(\frac{l}{t_1}+\frac{l}{t_2}\right) = \frac{l}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}\right)$

Speed of the stream = $\frac{1}{2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right) = \frac{l}{2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right)$

Example : 12 A boat covers 20 km in an hour with downstream and covers the same distance in 2 h with upstream. Then, find the speed of boat in still water and speed of stream.

Solution: Here, l=20 km, $t_1 = 1\ h, t_2 = 2\ h$

Speed of boat in still water = $\frac{l}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}\right) = \frac{20}{2}\left(\frac{1}{1}+\frac{1}{2}\right) = 10 \times \frac{3}{2} = 15\ km/\text{h}$

Speed of stream== $\frac{l}{2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right) = \frac{20}{2}\left(\frac{1}{1}-\frac{1}{2}\right) = 10 \times \frac{1}{2} = 5\ km/\text{h}$