Continuous Random Variable

I. A continuous random variable X has PDF $f(x) = K$, $0 \le x \le 1$. Find constant K & $P(X \le \frac{1}{4})$.

Soln.

i). $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^1 K\,dx = 1$$

$$K(x)_0^1 = 1$$

$$K(1-0) = 1$$

$$K = 1$$

$$\therefore f(x) = 1, \quad 0 \le x \le 1$$

ii). $P(X \le \frac{1}{4})$

$$= \int_0^{1/4} f(x)\,dx$$

$$= \int_0^{1/4} dx = (x)_0^{1/4} = \frac{1}{4}$$

1]. Test whether $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$ can be the probability density function of a continuous random variable.

Soln.

To prove $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Now,

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} e^{-x}\,dx$$

$$= \left[-e^{-x}\right]_0^{\infty}$$

$$= -e^{-\infty} + e^{0}$$

$$= 0 + 1$$

$$= 1$$

$\therefore$ $f(x)$ is probability density function.

3]. A continuous Random variable $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ is a pdf and find

i). $P(x < \frac{1}{2})$

ii). $P(1/4 < x < 1/2)$

iii) $P(x > 3/4 \,/\, x > 1/2)$

Soln.

i). $P(x < 1/2) = \int_0^{1/2} 2x\,dx$

$$= \left[2\frac{x^2}{2}\right]_0^{1/2}$$

$$= (1/2)^2 - 0 = 1/4$$

ii). $P\left(\frac{1}{4} < x < \frac{1}{2}\right)$

$$= \int_{1/4}^{1/2} 2x \, dx$$

$$= \left[2\frac{x^2}{2}\right]_{1/4}^{1/2}$$

$$= \frac{1}{4} - \frac{1}{16} = \frac{4-1}{16}$$

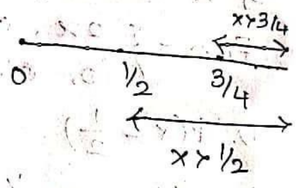
$$= \frac{3}{16}$$

iii). $P\left(x > \frac{3}{4} \,/\, x > \frac{1}{2}\right)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\therefore P\left(x > 3/4 \,/\, x > 1/2\right) = \frac{P\left(x > 3/4 \cap x > 1/2\right)}{P(x > 1/2)}$$

$$= \frac{P(x > 3/4)}{P(x > 1/2)}$$



$$= \frac{\int_{3/4}^{1} 2x \, dx}{\int_{1/2}^{1} 2x \, dx}$$

$$= \frac{\left(2\frac{x^2}{2}\right)_{3/4}^{1}}{\left(2\frac{x^2}{2}\right)_{1/2}^{1}} = \frac{1 - \frac{9}{16}}{1 - \frac{1}{4}} = \frac{\frac{16-9}{16}}{\frac{4-1}{4}}$$

$$= \frac{7}{16} \cdot \frac{4}{3}$$

$$= \frac{7}{12}$$

Hw 1]. If $f(x) = \begin{cases} C(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$

i). Find C ii). $P(X > 1)$

2]. Test whether

$f(x) = \begin{cases} x^2/3, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$ is a pdf.

3]. Test whether

$f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ is a pdf.

4]. If $f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ 3a - ax, & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$

Find i). a

ii). $P(X \le 1.5)$

iii). Distribution function.

Soln.

i). WKT $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (3a - ax)\,dx = 1$$

$$\left[\frac{ax^2}{2}\right]_0^1 + [ax]_1^2 + \left[3ax - a\frac{x^2}{2}\right]_2^3 = 1$$

$$\frac{a}{2}(1-0) + a(2-1) + \left(3a(3) - a\frac{9}{2}\right) - \left(3a(2) - a\frac{4}{2}\right) = 1$$

$$\frac{a}{2} + a + \left[9a - \frac{9a}{2} - 6a + 2a\right] = 1$$

$$6a - \frac{8a}{2} = 1$$

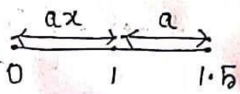
$$6a - 4a = 1$$

$$2a = 1$$

$$a = 1/2$$

ii). $P(X \leq 1.5)$

$$= \int_0^1 ax\,dx + \int_1^{1.5} a\,dx$$



$$= \left(\frac{ax^2}{2}\right)_0^1 + (ax)_1^{1.5}$$

$$= \frac{a}{2}(1-0) + a(1.5-1)$$

$$= \frac{a}{2} + 0.5a = \frac{a}{2} + \frac{a}{2}$$

$$= a$$

$$= 1/2$$

iii). Distribution Function:

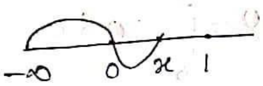
$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

For $x \leq 0$

$$F(x) = 0$$

for $0 \le x \le 1$

$$F(x) = \int_{-\infty}^{x} f(x)\, dx$$



$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\, dx$$

$$= \int_{0}^{x} ax\, dx$$

$$= \left(a\frac{x^2}{2}\right)_0^x$$

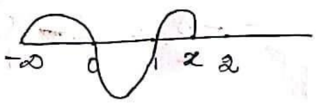
$$= \frac{a}{2}(x^2 - 0)$$

$$= \frac{x^2}{4}$$

For $1 \le x \le 2$

$$F(x) = \int_{-\infty}^{x} f(x)\, dx$$

$$= \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{x} f(x)\, dx$$



$$= 0 + \int_{0}^{1} ax\, dx + \int_{1}^{x} a\, dx$$

$$= \left(\frac{ax^2}{2}\right)_0^1 + (ax)_1^x$$

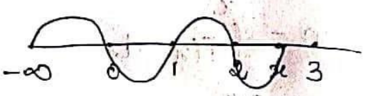
$$= \frac{a}{2}(1-0) + a(x-1)$$

$$= \frac{1}{4} + \frac{x}{2} - \frac{1}{2}$$

$$= \frac{-1}{4} + \frac{x}{2}$$

For $2 \le x \le 3$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$



$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{x} f(x)\,dx$$

$$= 0 + \int_{0}^{1} ax\,dx + \int_{1}^{2} a\,dx + \int_{2}^{x} (3a - ax)\,dx$$

$$= \left(\frac{ax^2}{2}\right)_0^1 + (ax)_1^2 + \left(3ax - a\frac{x^2}{2}\right)_2^x$$

$$= \frac{a}{2}(1-0) + a(2-1) + \left(3ax - a\frac{x^2}{2}\right) - \left(3a(2) - \frac{a}{2}(4)\right)$$

$$= \frac{a}{2} + a + 3ax - \frac{ax^2}{2} - 6a + 2a$$

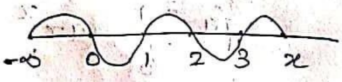
$$= \frac{a}{2} - 3a + 3ax - \frac{ax^2}{2}$$

$$= -\frac{5a}{2} + 3ax - \frac{ax^2}{2}$$

$$= -\frac{5}{4} + \frac{3x}{2} - \frac{x^2}{4}$$

For $x > 3$,

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$



$$F(x) = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx + \int_{3}^{x} f(x)\,dx$$

$$= 0 + \int_{0}^{1} ax\,dx + \int_{1}^{2} a\,dx + \int_{2}^{3} (3a - ax)\,dx + 0$$

$$= \left(\frac{ax^2}{2}\right)_0^1 + (ax)_1^2 + \left(3ax - a\frac{x^2}{2}\right)_2^3$$

$$= \frac{a}{2}(1-0) + a(2-1) + \left(3a(3) - \frac{a}{2}9\right) - \left(3a(2) - \frac{a}{2}(4)\right)$$

$$= \frac{a}{2} + a + 9a - \frac{9}{2}a - 6a + 2a$$

$$= 6a - \frac{8a}{2}$$

$$= \frac{6}{2} - \frac{4}{2}$$

$$= 3 - 2$$

$$F(x) = 1$$

$$\therefore \quad F(x) = \begin{cases} 0, & x \le 0 \\ x^2/4, & 0 \le x \le 1 \\ -1/4 + x/2, & 1 \le x \le 2 \\ -5/4 + 3x/2 - x^2/4, & 2 \le x \le 3 \\ 1, & x \ge 3 \end{cases}$$

CS Scanned with CamScanner