Subject Code & Name**: 19AST203 Aircraft Structural Mechanics**

TOPIC: **Strain Energy in shear loadings**

2.1.3 Strain energy in Pure shear loading

Let us consider a section subjected to shearing force Q acting across two of its opposite forces (Figure 2.4). The workdone by Q in a shear displacement of element 'dx' is given by,

$$U = \frac{1}{2} \times Q \times (\text{Shear displacement of element dx})$$

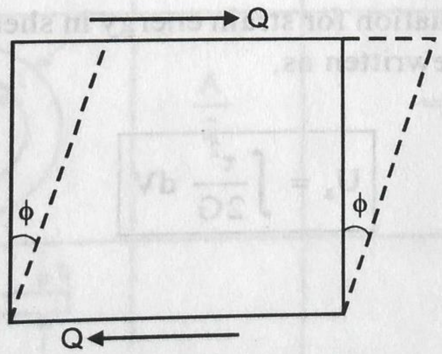

Figure 2.4 Shear loaded structure

$$\text{Shear displacement, dx} = \frac{\tau}{G}x$$

$$dx = \frac{Q}{A_r} \frac{x}{G}$$

Where, A_r → Reduced area

∴ Strain energy, $U = \frac{1}{2} \times Q \times \frac{Qx}{A_r\, G}$

$$U = \frac{Q^2}{2A_r\, G}\, x$$

The total strain energy for any small section is,

$$dU = \frac{Q^2}{2A_r\, G}\, dx$$

Integrating above equation,

$$\boxed{U_s = \int \frac{Q^2}{2A_r G}\, dx} \quad \text{... (2.5)}$$

This is the equation for strain energy in shear loading structure and also it can be rewritten as,

$$\boxed{U_s = \int \frac{\tau^2}{2G}\, dV} \quad \text{... (2.6)}$$

2.1.4 Strain energy in torsional loading

If T is the twisting moment acting on element dx of a structural member (Figure 2.5), the workdone is given by,

$$W = \frac{1}{2} T \times (\text{Torsional displacement of element dx})$$

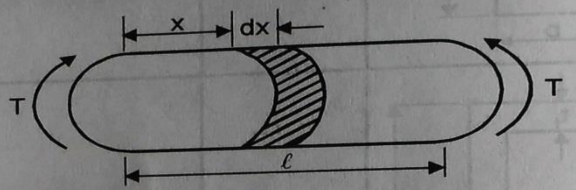

Figure 2.5: Torsional loaded structure

$$\therefore \quad \text{The strain energy, } U = \frac{1}{2} \times T \times \frac{Tx}{KG}$$

Where, K - Torsional constant of cross-section

$$U = \frac{1}{2}\frac{T^2}{KG}x$$

The strain energy on the small element is given by,

$$dU = \frac{T^2}{2KG}dx$$

Integrating this equation,

$$\boxed{U_T = \int \frac{T^2}{2KG}dx} \quad \text{... (2.7)}$$

This is the equation for strain energy in torsional loading structures in a general form.

Note:

In general the strain energy of a member in following cases have to be remembered for solving problems.

1) **Rigid-Jointed plane frame**

$$U = \left[\int \frac{P^2}{2AE} + \int \frac{M^2}{2EI} + \int \frac{Q^2}{2A_r \cdot G}\right] dx \qquad \text{... (2.8)}$$

2) **Rigid-Jointed space frame**

$$U = \left[\int \frac{P^2}{2AE} + \int \frac{M^2}{2EI} + \int \frac{Q^2}{2A_r \cdot G} + \int \frac{T^2}{2KG}\right] dx \qquad \text{.. (2.9)}$$

3) **Rigid-jointed space frame subjected to biaxial shear forces (Q_x and Q_y) and biaxial moments (M_x and M_y)**

$$U = \int \frac{P^2}{2AE} \cdot dx + \int \frac{M_x^2}{2EI_{xx}} \cdot dx + \int \frac{M_y^2}{2EI_{yy}} \cdot dy \qquad \text{...(2.10)}$$

$$+ \int \frac{Q_x^2}{2A_{r_x} G} \cdot dx + \int \frac{Q_y^2}{2A_{r_y} G} \cdot dx + \int \frac{T^2}{2KG} \cdot dx$$

Where A_{r_x}, A_{r_y} - Reduced areas of cross-section when the shear occurs in xz and yz planes respectively.

2.2 CASTIGLIANO'S THEOREMS

2.2.1 Theorem-I

In any structure subjected to any load system, the deflection at any point is given by the partial differential coefficient of the total strain energy stored with respect to force acting at a point.

i.e., $$\boxed{x_i = \frac{\partial U}{\partial P_i} \text{ and } \theta_i = \frac{\partial U}{\partial M_i}}$$