



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

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UNIT-I Vector calculus

Gradient and directional derivative

unit-1

Scalar quantities

A scalar quantity is that which has only magnitude and it is not related to any direction

Vector quantities

A vector quantity is that which has both magnitude and direction

Vector differential operators

The vector differential operator is denoted by ∇ and it is defined by $\nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}$

Note

- * $\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$
- * $\vec{i} \times \vec{j} = \vec{j} \times \vec{k} = \vec{k} \times \vec{i} = 1$

Gradient of a scalar point function

If $\phi(x, y, z)$ is a scalar point function and it is continuously differentiable then it is defined as

$$\nabla \phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$$

as grad (ϕ) or $\nabla \phi$.



Problem 1

Find the gradient $\nabla\phi$, where $\phi = x^2 + y^2 + z^2$

$$\nabla\phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$$

$$\begin{aligned}\text{using chain rule method of partial derivative, } \\ \nabla\phi &= \vec{i} \frac{\partial}{\partial x} (x^2 + y^2 + z^2) + \vec{j} \frac{\partial}{\partial y} (x^2 + y^2 + z^2) + \vec{k} \frac{\partial}{\partial z} (x^2 + y^2 + z^2) \\ &= \vec{i}(2x) + \vec{j}(2y) + \vec{k}(2z) \quad (\text{by } \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})\end{aligned}$$

$$\nabla\phi = x\vec{i} + y\vec{j} + z\vec{k} \quad \text{using formula}$$

Problem 2

Find $\nabla\phi$ where $\phi = 3x^2y - y^3z^2$, at $(1,1,1)$

$$\begin{aligned}\text{using chain rule method of partial derivative, } \\ \nabla\phi &= \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z} \\ &= \vec{i} \frac{\partial}{\partial x} (3x^2y - y^3z^2) + \vec{j} \frac{\partial}{\partial y} (3x^2y - y^3z^2) + \\ &\quad \vec{k} \frac{\partial}{\partial z} (3x^2y - y^3z^2) \\ &= \vec{i}(6xy) + \vec{j}(3x^2 - 3y^2z^2) + \vec{k}(-2y^3z)\end{aligned}$$

$$\nabla\phi(1,1,1) = 6\vec{i} + 0\vec{j} - 2\vec{k}$$

$$\nabla\phi(1,1,1) = 3\vec{i} + 0\vec{j} - \vec{k}$$

$$\text{by using formula } \nabla\phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z} = \phi \nabla$$



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Problem 2

Find the directional derivative of $4xz^2 + xy^2$ at the point $(1, -2, 1)$ in the direction $2\vec{i} - \vec{j} - 2\vec{k}$.

The directional derivative is $\frac{\nabla \Phi \cdot \vec{a}}{|\vec{a}|}$

$$\nabla \Phi = \vec{i}(4z^2 + y^2) + \vec{j}(x^2) + \vec{k}(8xz + 2xy)$$

$$\nabla \Phi = 2\vec{i} + \vec{j} + 6\vec{k}$$

$(1, -2, 1)$

$$\frac{2}{\sqrt{4+1+4}} + \frac{1}{\sqrt{4+1+4}} + \frac{6}{\sqrt{4+1+4}} = \Phi$$

$$|\vec{a}| = \sqrt{4+1+4}$$

$$|\vec{a}| = 3.$$

$$\therefore \text{the directional derivative is } \frac{(2\vec{i} + \vec{j} + 6\vec{k}) \cdot (2\vec{i} - \vec{j} - 2\vec{k})}{3}$$