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Unit - I

Vector calculus

Scalar quantities

A scalar quantity is that which has only magnitude and it is not related to any direction

Vector quantities

A vector quantity is that which has both magnitude and direction

Vector differential operator

The vector differential operator is denoted by ∇ and it is defined by $\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$

Note

* $\vec{i}\times\vec{i} = \vec{j}\times\vec{j} = \vec{k}\times\vec{k} = 1$
* $\vec{i}\times\vec{j} = \vec{j}\times\vec{k} = \vec{k}\times\vec{i} = 0$

Gradient of a scalar point function

If $\phi(x,y,z)$ is a scalar point function and it is continuously differentiable then it is defined as

$$\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$$

It is denoted as grad (ϕ) (or) $\nabla\phi$.

Problem 1

Find the gradient ϕ, where $\phi = x^2+y^2+z^2$

$$\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$$

$$= \vec{i}\frac{\partial}{\partial x}(x^2+y^2+z^2) + \vec{j}\frac{\partial}{\partial y}(x^2+y^2+z^2) + \vec{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)$$

$$= \vec{i}(2x)+\vec{j}(2y)+\vec{k}(2z) \quad (\text{by} \div 2).$$

$$\nabla\phi = x\vec{i}+y\vec{j}+z\vec{k}$$

Problem 2

Find $\nabla\phi$ where $\phi = 3x^2y - y^3z^2$ at $(1,1,1)$

$$\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$$

$$= \vec{i}\frac{\partial}{\partial x}(3x^2y-y^3z^2) + \vec{j}\frac{\partial}{\partial y}(3x^2y-y^3z^2) + \vec{k}\frac{\partial}{\partial z}(3x^2y-y^3z^2)$$

$$= \vec{i}(6xy) + \vec{j}(3x^2-3y^2z^2) + \vec{k}(-2y^3z)$$

$$\nabla\phi_{(1,1,1)} = 6\vec{i}+0\vec{j}-2\vec{k}$$

$$\nabla\phi_{(1,1,1)} = 3\vec{i}+0\vec{j}-\vec{k}$$

Problem 2

Find the directional derivative of $4xz^2 + xyz$ at the point $(1,-2,1)$ in the direction $2\vec{i} - \vec{j} - 2\vec{k}$

The directional derivative is $\dfrac{\nabla\phi \cdot \vec{a}}{|\vec{a}|}$

$$\nabla\phi = \vec{i}(4z^2 + yz) + \vec{j}(xz) + \vec{k}(8xz + xy)$$

$$\nabla\phi_{(1,-2,1)} = 2\vec{i} + \vec{j} + 6\vec{k}$$

$$|\vec{a}| = \sqrt{4+1+4}$$

$$|\vec{a}| = 3.$$

$\therefore$ The directional derivative is $= \dfrac{(2\vec{i}+\vec{j}+6\vec{k})\cdot(2\vec{i}-\vec{j}-2\vec{k})}{3}$