



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Higher order linear differential equations with constant coefficients

Type : I

$R(x) = e^{ax} \therefore$ Rule : Replace $D = a$

Example 1 :

Solve $\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right) + y = 2e^x$

Soln: Given,

$$\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right) + y = 2e^x$$

$$(D^2y - 2Dy + y) = 2e^x \quad (D^2 - 2D + 1)y = 2e^x$$

$$(D^2 - 2D + 1)y = 2e^x$$

The A.E. is

$$m^2 - 2m + 1 = 0$$

$$(m-1)(m-1) = 0$$

$$m=1, m=1$$

$$C.F. = (Ax + B)e^x$$

$$P.I. = \frac{1}{F(D)} R(x)$$

$$= \frac{1}{D^2 - 2D + 1} 2e^x$$

$$= \frac{2}{1^2 - 2(1) + 1} e^x$$

$$= \frac{2}{0} e^x = \frac{2}{0} e^x$$

\therefore The A.E. is

$$m^2 - 2m + 1 = 0$$

$$(m-1)(m-1) = 0$$

$$m_1 = m_2 = 1$$

$$C.F. = (Ax + B)e^x$$

$$P.I. = \frac{1}{D^2 - 2D + 1} e^{2x}$$

$$= \frac{1}{2^2 - 2(2) + 1} e^{2x}$$

$$P.I. = e^{2x}$$

$$y = (Ax + B)e^x + e^{2x}$$



$$P \cdot I_1 = \frac{x}{f'(0)} R(x)$$

$$= \frac{2x}{2D-2} e^x$$

$$= \frac{2x}{0} e^x$$

$$P \cdot I_2 = \frac{x^2}{f''(0)} e^{2x}$$

$$= \frac{2x^2}{2} e^{2x}$$

$$P \cdot I = x^2 e^x$$

$$Y = C.F + P \cdot I$$

$$y = (Ax+B)e^x + x^2 e^x$$

Example: 2.

Solve $(D^2 - 2D + 1)y = \cosh x$.

Soln: Given, $(D^2 - 2D + 1)y = \frac{e^x + e^{-x}}{2}$

$$(D^2 - 2D + 1)y = \cosh x$$

The A.E is $m^2 - 2m + 1 = 0$

$$m=1, m=1$$

$$C.F = (Ax+B)e^x$$

$$P \cdot I = \frac{1}{f(0)} R(x)$$

$$= \frac{1}{D^2 - 2D + 1} \cosh x$$

$$= \frac{1}{D^2 - 2D + 1} \left[\frac{e^x + e^{-x}}{2} \right]$$

$$= \frac{1}{2} \left[\frac{1}{D^2 - 2D + 1} e^x + \frac{1}{D^2 - 2D + 1} e^{-x} \right]$$

$$= \frac{1}{2} \left[\frac{1}{1-2+1} e^x + \frac{1}{1+2+1} e^{-x} \right]$$



$$\begin{aligned} &= \frac{1}{2} \left[\frac{1}{0} e^x + \frac{1}{4} e^{-x} \right] & D=a=1 \\ &= \frac{1}{2} \left[\frac{x}{2D-2} e^x + \frac{1}{4} e^{-x} \right] & D=a=-1 \\ &= \frac{1}{2} \left[\frac{x}{0} e^x + \frac{1}{4} e^{-x} \right] & D=a=1 \\ &= \frac{1}{2} \left[\frac{x^2}{2} e^x + \frac{1}{4} e^{-x} \right] \\ &= \frac{1}{4} \left[x^2 e^x + \frac{e^{-x}}{2} \right] \\ &= \frac{1}{4} \left[\frac{2x^2 e^x + e^{-x}}{2} \right] \\ P.I &= \frac{1}{8} [2x^2 e^x + e^{-x}] \\ Y &= C.P + P.I. \\ Y &= (Ax + B)e^x + \frac{1}{8} [2x^2 e^x + e^{-x}] \end{aligned}$$