Type : I

$R(x) = e^{ax}$ $\therefore$ Rule : Replace $D = a$

Example 1 :

Solve $\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right) + y = 2e^x$

Soln: Given,

$$\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right) + y = 2e^x$$

$$(D^2y - 2Dy + y) = 2e^x$$

$$(D^2 - 2D + 1)y = 2e^x$$

The A.E is

$$m^2 - 2m + 1 = 0$$

$$(m-1)(m-1) = 0$$

$$m = 1, \quad m = 1$$

$$C.F = (Ax + B)e^x$$

$$P.I = \frac{1}{F(D)} R(x)$$

$$= \frac{1}{D^2 - 2D + 1} 2e^x$$

$$= \frac{2}{1^2 - 2(1) + 1} e^x$$

$$= \frac{2}{0} e^x$$

$$\frac{1}{1+0+1} = \frac{2}{0} e^x \quad \frac{1}{0}$$

$(D^2 - 6D + 9)y = e^{2x}$

$\therefore$ The A.E is

$m^2 - 6m + 9 = 0.$

$(m-3)(m-3) = 0$

$m_1 = m_2 = 3.$

$C.F = (Ax + B)e^{3x}$

$$P.I = \frac{1}{D^2 - 6D + 9} e^{2x}$$

$$= \frac{1}{2^2 - 6(2) + 9} e^{2x}$$

$P.I = e^{2x}$

$y = (Ax + B)e^{3x} + e^{2x}$

$$P.I_1 = \frac{x}{f'(D)} R(x)$$

$$= \frac{2x}{2D-2} e^x$$

$$\frac{2x}{0} e^x$$

$$P.I_2 = \frac{x^2}{f''(D)} e^x(x)$$

$$= \frac{2x^2}{2} e^x$$

$$P.I = x^2 e^x$$

$$Y = C.F + P.I$$

$$y = (Ax+B)e^x + x^2 e^x .$$

Example: 2.

Solve $(D^2 - 2D + 1)y = \cosh x$.

Soln: Given, $\frac{e^{2x} - e^{-x}}{2}$

$$(D^2 - 2D + 1)\, y = \cosh x.$$

The A.E is $m^2 - 2m + 1 = 0$

$$m = 1, \quad m = 1.$$

$\frac{e^x + e^{-x}}{2}$

$$C.F = (Ax + B)e^x$$

$$P.I = \frac{1}{f(D)} R(x)$$

$$= \frac{1}{D^2 - 2D + 1} \cosh x$$

$$= \frac{1}{D^2 - 2D + 1} \left[\frac{e^x + e^{-x}}{2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{D^2 - 2D + 1} e^x + \frac{1}{D^2 - 2D + 1} e^{-x}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-2+1} e^x + \frac{1}{1+2+1} e^{-x}\right]$$

$$= \frac{1}{2}\left[\frac{1}{0}e^{x} + \frac{1}{4}e^{-x}\right] \qquad D=a=1,\ D=a=-1,\ D=a=1$$

$$= \frac{1}{2}\left[\frac{x}{2D-2}e^{x} + \frac{1}{4}e^{-x}\right]$$

$$= \frac{1}{2}\left[\frac{x}{0}e^{x} + \frac{1}{4}e^{-x}\right]$$

$$= \frac{1}{2}\left[\frac{x^2}{2}e^{x} + \frac{1}{4}e^{-x}\right]$$

$$= \frac{1}{4}\left[x^2e^{x} + \frac{e^{-x}}{2}\right]$$

$$= \frac{1}{4}\left[\frac{2x^2e^{x} + e^{-x}}{2}\right]$$

$$P.I = \frac{1}{8}\left[2x^2e^{x} + e^{-x}\right]$$

$$Y = C.F + P.I.$$

$$Y = (Ax + B)e^{x} + \frac{1}{8}\left[2x^2e^{x} + e^{-x}\right]$$