



DEPARTMENT OF MATHEMATICS

UNIT - II

VECTOR CALCULUS

Scalar Quantity :

A scalar quantity is that which has magnitude and is not related to any direction.

Vector Quantity :

A vector quantity is that which has both magnitude and direction.

Scalar point function :

If corresponding to each point P of a region R there corresponds a scalar denoted by $\phi(P)$ or $\phi(x, y, z)$ then ϕ is said to be a scalar point function for the region R .

Example : The temperature $\phi(P)$ at any point P of a body occupying a certain region is a scalar point function.

Vector point function :

If corresponding to each point P of a region R , there corresponds a vector denoted by $F(P)$, then F is said to be a vector point function for the region R .

Example : The acceleration $F(P)$ of a particle at any time t occupying the position P in a certain region is a vector point function.

$$\vec{u} \pm \vec{v} = (\vec{u} \pm \vec{v})$$
$$F \text{ (vec)} \pm G \text{ (vec)} = (F \pm G) \text{ (vec)}$$



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Vector Differential Operator:

The vector differential operator ∇ is defined as,

$$\nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} = \pm \vec{i} \frac{\partial}{\partial x}$$

Gradient of a scalar point function:

Let $\phi(x, y, z)$ be a scalar point function and is continuously differentiable then the vector,

$$\begin{aligned} \nabla \phi &= \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \phi \\ &= \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z} \end{aligned}$$

is called the gradient of ϕ and is written as $\text{grad } \phi$.

i.e., $\boxed{\text{grad } \phi = \nabla \phi}$

Note:

1. $\nabla \phi$ defines a vector field.
2. $\nabla \phi \neq \phi \nabla$. There will be no '•' or 'x' between ϕ and ∇ .

Properties of Gradient:

1. If f and g are two scalar point functions then,

$$\nabla(f \pm g) = \nabla f \pm \nabla g$$

(or) $\text{grad}(f \pm g) = \text{grad } f \pm \text{grad } g$



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(2) If f and g are two scalar point functions then,

$$\nabla(fg) = f \nabla g + g \nabla f$$

(or) $\text{grad}(fg) = f(\text{grad } g) + g(\text{grad } f)$

(3) If f and g are two scalar point functions then,

$$\nabla\left(\frac{f}{g}\right) = \frac{g \nabla f - f \nabla g}{g^2} \text{ where } g \neq 0$$

(or) $\text{grad}\left(\frac{f}{g}\right) = \frac{g(\text{grad } f) - f(\text{grad } g)}{g^2}$

(4) Gradient of a constant is zero.

i.e., $\nabla \phi = 0$

Problems :

① Find $\text{grad } \phi$ where $\phi = x^2 + y^2 + z^2$.

Soln :

$$\begin{aligned} \nabla \phi &= \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z} \\ &= \vec{i} \frac{\partial}{\partial x} (x^2 + y^2 + z^2) + \vec{j} \frac{\partial}{\partial y} (x^2 + y^2 + z^2) + \vec{k} \frac{\partial}{\partial z} (x^2 + y^2 + z^2) \\ &= \vec{i} (2x) + \vec{j} (2y) + \vec{k} (2z) \\ \nabla \phi &= 2x\vec{i} + 2y\vec{j} + 2z\vec{k} \end{aligned}$$

② Find $\text{grad } \phi$ if $\phi = xyz$ at $(1, 1, 1)$

Soln :

$$\nabla \phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$$

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$$\begin{aligned}\nabla\phi &= \vec{i} \frac{\partial}{\partial x}(xyz) + \vec{j} \frac{\partial}{\partial y}(xyz) + \vec{k} \frac{\partial}{\partial z}(xyz) \\ &= \vec{i} yz + \vec{j} (xz) + \vec{k} (xy)\end{aligned}$$

$$\nabla\phi_{(1,1,1)} = \vec{i} + \vec{j} + \vec{k}$$

- ③ Find grad ϕ where $\phi = 3x^2y - y^3z^2$ at $(1,1,1)$.

soln:

$$\begin{aligned}\nabla\phi &= \vec{i} \frac{\partial\phi}{\partial x} + \vec{j} \frac{\partial\phi}{\partial y} + \vec{k} \frac{\partial\phi}{\partial z} \\ &= \vec{i} \frac{\partial}{\partial x}(3x^2y - y^3z^2) + \vec{j} \frac{\partial}{\partial y}(3x^2y - y^3z^2) \\ &\quad + \vec{k} \frac{\partial}{\partial z}(3x^2y - y^3z^2) \\ &= \vec{i} (6xy) + \vec{j} (3x^2 - 3y^3z^2) + \vec{k} (-2y^3z)\end{aligned}$$

$$\nabla\phi_{(1,1,1)} = 6\vec{i} - 2\vec{k}$$

- ④ If $\phi = \log(x^2 + y^2 + z^2)$ find $\nabla\phi$.

soln:

$$\begin{aligned}\nabla\phi &= \vec{i} \frac{\partial\phi}{\partial x} + \vec{j} \frac{\partial\phi}{\partial y} + \vec{k} \frac{\partial\phi}{\partial z} \\ &= \vec{i} \frac{\partial}{\partial x} [\log(x^2 + y^2 + z^2)] + \vec{j} \frac{\partial}{\partial y} [\log(x^2 + y^2 + z^2)] \\ &\quad + \vec{k} \frac{\partial}{\partial z} [\log(x^2 + y^2 + z^2)] \\ &= \vec{i} \frac{1}{x^2 + y^2 + z^2} (2x) + \vec{j} \frac{1}{x^2 + y^2 + z^2} (2y) + \\ &\quad \vec{k} \frac{1}{x^2 + y^2 + z^2} (2z) \\ &= \frac{2}{x^2 + y^2 + z^2} (x\vec{i} + y\vec{j} + z\vec{k})\end{aligned}$$

