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#### **DEPARTMENT OF MATHEMATICS**

# LINE INTEGRALS: Suppose c is an arc and ? = xi + yj + zk is the position vector of any point P(x, y, z) on it and f is a vector point function at P. Then $\int \vec{f} \cdot d\vec{r}$ is called a line integral of $\vec{f}$ over C. Line integral $\int \vec{F} \cdot d\vec{r}$ is also known as the Lotal work done by the force F during a displacement from A to B. 1) Evaluate IF. dr where F = x2 y2 i+ yj and the curve c is y = 4x in the xy-plane from (0,0) to (4,4) 8 = xi+4i $d\vec{r} = dx \vec{i} + dy \vec{j}$ Given: F = x2 y2 i+ yj $\vec{F} \cdot d\vec{r} = (x^2 y^2 \vec{i} + y \vec{j}) \cdot (dx \vec{i} + dy \vec{j})$ $= x^2 y^2 dx + y dy visovenes o a$ Given: 42 = 47 $2y \, dy = 4 \, dx$ $y \, dy = 2 \, dx$ $\vec{F} \cdot d\vec{r} = x^{2}y^{2}dx + 2 dx = x^{2}(4x) dx + 2 dx$ $\int \vec{F} \cdot d\vec{r} = \int (4x^{3} + 2) dx$





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$$\int_{C} \vec{F} \cdot d\vec{r} = \int_{C}^{4} (4x^{3} + 2) dx$$

$$= \left[ \frac{1}{4} + \frac{1}{4} + 2x \right]_{0}^{4}$$

$$= \frac{1}{4} + 8 = 356 + 8$$

$$= 364$$
② If  $\vec{F} = x^{2} \vec{i} + xy \vec{j}$  evaluate  $\int_{C} \vec{F} \cdot d\vec{r}$  along the straight line  $y = x$  from  $(0,0)$  to  $(1,1)$ .

Soln:  $2/3$ .
③ If  $\vec{F} = 5xy \vec{i} + 2y \vec{j}$ , evaluate  $\int_{C} \vec{F} \cdot d\vec{r}$  where  $C$  is the past of the curve  $y = x^{3}$  between  $x = 1$  and  $x = 2$ .

Soln:  $\vec{F} = 5xy \vec{i} + 2y \vec{j}$ 

$$d\vec{r} = dx \vec{i} + dy \vec{j}$$

$$\vec{F} \cdot d\vec{r} = 5xy dx + 2y dy$$
Given:  $y = x^{3} \Rightarrow dy = 5x^{2} dx$ 

$$\therefore \int_{C} \vec{F} \cdot d\vec{r} = \int_{C}^{3} 5x(x^{3}) dx + 2(x^{3}) 5x^{2} dx$$

$$= \int_{C}^{3} (5x^{4} + 6x^{5}) dx$$

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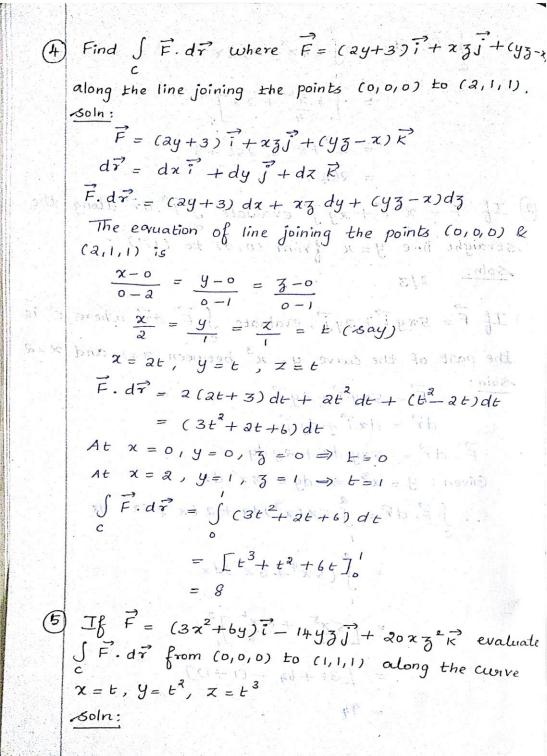
$$= \int_{C}^{3} (3x + 6x^{5}) dx$$





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DEPARIMENT OF MATHEMATICS

$$F = (3x^2 + 6y)\vec{i} - 14y\vec{j} + 30x\vec{j}^2 \vec{k},$$

$$d\vec{i} = dx\vec{i} + dy \vec{j} + dx\vec{k}$$

$$F \cdot d\vec{i} = (3x^2 + 6y)dx - 14y\vec{j} dy + 30x\vec{j}^2 dz$$
Given:  $x = t$ ,  $y = t^2$ ,  $z = t^3$ 

$$dx = dt$$
,  $dy = xtdt$ ,  $dz = 3t^2dt$ 

$$F \cdot d\vec{i} = (3t^2 + 6t^2)dt - 14(t^3 t^3)xtdt + 20(t t^6)3t^3dt$$

$$= (9t^2 - 28t^6 + 60t^9)dt$$

$$G = \frac{1}{3}(9t^2 - 28t^6 + 60t^9)$$



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