Problems Based on Trains:

Problems based on trains are same as the problems related to 'Speed, Time and Distance' and some concepts of 'Speed, Time and Distance' are also applicable to these problems. The only difference is that the length of the moving object (train) is taken into consideration in these types of problems.

Problems:

Rule :1

Speed of train (S) = $\frac{Distance\ covered\ (d)}{Time\ taken\ (t)}$ (or) $S = \frac{d}{t}$

Here, unit of speed is m/s or km/h.

(a) $a\ km/h = \left(a \times \frac{5}{18}\right) m/s$ (b) $a\ m/s = \left(a \times \frac{18}{5}\right) km/h$

Example: 1 Convert $360\ km/h$ into m/s.

Solution: $360\ km/h = 360 \times \frac{5}{18}\ m/s = 100$ m/s.

Example: 2 Convert $150\ m/s$ into km/h.

Solution: $150\ m/s = 150 \times \frac{18}{5}\ km/h = 30 \times 18\ km/h = 540\ km/h$.

Rule: 2

The distance covered by train in passing a pole or a standing man or a signal post or any other object (of negligible length) is equal to the length of the train.

Example: 3 A train covers 85 m in passing a signal post. What is the length of the train?

Solution: We know that, the distance covered by a train in passing a pole or a standing man or a signal post or any other subject (of negligible length) is equal to the length of the train. So, in this case, train covers 85 m to pass a signal post.

Rule: 3

If a train passes a stationary object (bridge, platform etc;) having some length, then the distance covered by train is equal to the sum of the lengths of train and that particular stationary object which it is passing.

Example: 4 A 29 m long train passes a platform which is 100 m long. Find the distance covered by the train in passing the platform.

Solution : We know that, when a train passes a stationary object having some length, then the distance covered by train is equal to the sum of the lengths of train and that particular stationary object. In this case, stationary object is 100m long platform.

Required distance = Length of train + Length of platform = 29 + 100 = 129 m.

Rule : 4

If two trains are moving in opposite directions, then their relative speed is equal to the sum of the speeds of both the trains.

Example : 5 Two trains are moving in opposite directions with speeds of 4 m/s and 8 m/s, respectively. Find their relative speed**.**

Solution: When two trains are moving in opposite direction, then their

Relative speed = Sum of the speeds of both the trains

Required relative speed = 4 m/s + 8 m/s = 12 m/s

Rule : 5

If two trains are moving in the same direction, then the relative speed is the difference of speeds of both trains.

Example : 6 Two trains are moving in the same direction with speeds of 19 km/h and 25 km/h, respectively. What will be the relative speed of the train running at 25 km/h in respect of the train running at 19 km/h?

Solution: We know that, when two trains are running in the same direction, then the relative speed is equal to the difference of speeds of both the trains.

Required relative speed = 25 km/h - 19 km/h = 6 km/h.

Rule : 6

If two trains of lengths x and y are moving in opposite directions with speeds of u and v respectively, then time taken by the trains to cross each other is equal to $\frac{(x+y)}{(u+v)}$.

Example : 7 Two trains of lengths 80 m and 90 m are moving in opposite directions at 10 m/s and 7 m/s, respectively. Find the time taken by the trains to cross each other.

Solution: According to the formula,

Required time = $\frac{x+y}{u+v}$

Where, $x = 80\,m, y = 90\,m, u = 10\,m/s$ and $v = 7\,m/s$

Required time = $\frac{80+90}{10+7} = \frac{170}{17} = 10\,s$.

Rule :7

If two trains of lengths x and y are moving in the same direction with speeds of u and v respectively, then time taken by the faster train to cross the slower train is equal to $\frac{(x+y)}{(u-v)}$.

Example : 8 Two trains of lengths 75 m and 95 m are moving in the same direction at 9 m/s and 8 m/s, respectively. Find the time taken by the faster train to cross the slower train.

Solution: According to the formula,

Required time = $\frac{x+y}{u-v}$

Where, $x = 75\,m, y = 95\,m, u = 9\,m/s$ and $v = 8\,m/s$

Required time = $\frac{75+95}{9-8} = 170\,s$.

Rule : 8

If two trains start at the same time from points P and Q towards each other and after crossing each other, they take t_1 and t_2 time in reaching points Q and P respectively, then

(P's speed) : (Q's speed) = $\sqrt{t_2} : \sqrt{t_1}$

Example : 9 Two trains start at the same time from points A and B towards each other and after crossing each other, they take 25 h and 9 h in reaching points B and A, respectively. Find the ration of speeds of 1st train to that of 2nd train.

Solution: Given that, $t_1 = 25\,h$ and $t_2 = 9\,h$

According to the formula,

(1st train's speed) : (2nd train's speed) = $\sqrt{t_2} : \sqrt{t_1} = \left(\sqrt{9} : \sqrt{25}\right) = 3:5$

Example : 10 A train passes a standing man in 6 s and a 210 m long platform in 16 s. Find the length and the speed of the train.

Solution : Let length of the train be L.

As the speed while crossing a standing man and while crossing the platform remains same.

According to the question, $\frac{L}{6} = \frac{L+210}{16}$ $\quad \left[Speed = \frac{Length}{Time}\right]$

16L = 6L+1260

10L = 1260

$$L = \frac{1260}{10} = 126\ m$$

Speed of the train = $\frac{Length}{Time} = \frac{126}{6} = 21\ m/s.$

Example : 11 A 250 m long train is running at 100 km/h. In what time, will it pass a man running at 10 km/h in the same direction in which the train is going?

Solution : Speed of the train relative to man = 100 – 10 = 90 km/h

$$= 90 \times \frac{5}{18}\ m/s = 5 \times 5 = 25\ m/s$$

Distance covered in passing the man = 250 m

Time taken = $\frac{250}{25} = 10\ s.$

Example : 12 A 220 m long train is running at 120 km/h. In what time, will it pass a man running in the direction opposite to that of the train at 12 km/h?

Solution: Speed of the train relative to man = (120 +12) km/h = 132 km/h

$$=\left(132 \times \frac{5}{18}\right)\ m/s = \frac{110}{3} m/s$$

Distance covered in passing the man = 220 m

Time taken = $\frac{220}{110} \times 3 = 2 \times 3 = 6\ s$

Fast Track Techniques:

Technique : 1

If a train of length L m passes a platform of x m in $t_1 s$, then time taken $t_2 s$ by the same train to pass a platform of length y m is given as $t_2 = \left(\frac{L+Y}{L+X}\right) t_1$

Example : 13 A train of length 250 m, passes a platform of 350 m length in 50 s. What time will this train take to pass the platform of 230 m length.

Solution: Here, L=250 m, $x = 350\ m, t_1 = 50s, y = 230m$ and t_2 =?

$$t_2 = \left(\frac{L + y}{L + x}\right)$$

$$t_1 = \left(\frac{250 + 230}{250 + 350}\right) \times 50 = \frac{480}{600} \times 50 = 40\ s$$

Technique: 2

From stations P and Q, two trains start moving towards each other with the speeds of a and b, respectively. When they meet each other, it is found that one train covers distance d more than that of another train. In such cases, distance between stations P and Q is given as

$$\left(\frac{a+b}{a-b}\right) \times d.$$

Example : 14 From stations A and B, two trains start moving towards each other with the speeds of 150 km/h and 130 km/h, respectively. When the two trains meet each other, it is found that one train covers 20 km more than that of another train. Find the distance between stations A and B.

Solution: Here, $= 150\ km/h$, $b = 130\ km/h$ and $d = 20\ km$

According to the formula,

Distance between stations A and B = $\left(\frac{a+b}{a-b}\right) \times d$

$= \left(\frac{150+130}{150-130}\right) \times 20 = \frac{280}{20} \times 20 = 280\ km$

Technique: 3

If two trains leave P for Q at time t_1 and t_2 and travel with speeds a and b respectively, then the distance d from P, where the two trains meet, is given as

$$d = \text{Difference in time} \times \frac{\text{Product of speeds}}{\text{Difference in speeds}} = (t_2 - t_1) \times \frac{a \times b}{b - a}$$

Where, $t_2 > t_1$ and $b > a$.

Example: 15 Two trains leave Patna for Delhi at 10:00 am and 10:30 am respectively and travel at 120 km/h and 150 km/h, respectively. How many kilometers from Patna, will the two trains meet?

Solution: Speed of 1st train = 120 km/h and Speed of 2nd train = 150 km/h

Relative speed of 2nd train = 150 – 120 = 30 km/h

As 1st train leaves $\frac{1}{2}h$ before, hence in $\frac{1}{2}h$, it will cover 60 km as it moves 120 km in 1 h.

Time taken by 2nd train to gain 60 km = $\frac{60}{30} = 2\ h$

Actual distance covered by 2nd train in 2 h = 2 ×150 = 300 km

Technique : 4

If a train overtakes two persons who are walking with speeds of a and b respectively, in the same direction and passes the two persons completely in t_1 and t_2 time, respectively, then

Length of the train = $\frac{\text{Difference in speeds} \times t_1 \times t_2}{(t_2 - t_1)}$, where $t_2 > t_1$

Note : In case $t_1 > t_2, (t_1 - t_2)$ is taken in place of $(t_2 - t_1)$ in the denominator.

Example : 16 A train overtakes two persons who are walking at the rate of 4 km/h and 8 km/h in the same direction and passes them completely in 18 and 20 s respectively. Find the length of the train.

Solution: Here, $t_1 = 18\ s, t_2 = 20\ s, a = 4\ km/h = 4 \times \frac{5}{18}\ m/s = \frac{10}{9}\ m/s$

$b = 8\ km/h = 8 \times \frac{5}{18}\ m/s = \frac{20}{9}\ m/s$

According to the formula,

Length of the train = $\frac{\text{Difference in speeds} \times t_1 \times t_2}{t_2 - t_1}$

$= \frac{\left(\frac{20}{9} - \frac{10}{9}\right) \times 18 \times 20}{20 - 18} = \frac{\frac{10}{9} \times 18 \times 20}{2} = 10 \times 20 = 200\ m$

Technique : 5

If two trains A and B start from stations/points P and Q towards Q and P respectively and after passing each other, they take t_1 and t_2 time to reach Q and P respectively and speed of train A is given as a, then

$$\text{Speed of train B} = a\sqrt{\frac{t_1}{t_2}}$$

Example:17 Two trains x and y start from Mumbai and Delhi towards Delhi and Mumbai, respectively. After passing each other, they take 12 h 30 min and 8 h to reach Delhi and Mumbai, respectively. If the train from Mumbai is moving at 60 km/h, then find the speed of the other train.

Solution: Here, a = 600 km/h, $t_1 = 12\ h\ and\ 30\ min = 12 + \frac{30}{60} = \frac{25}{2}\ h, t_2 = 8\ h$

According to the formula,

Speed of $y = a\sqrt{\frac{t_1}{t_2}} = 60 \times \sqrt{\frac{25}{2} \times 8} = 60 \times \frac{5}{4}\ km/h$ = 75 km/h

Technique : 6

The distance between P and Q is d km. A train with a km/h starts from station P towards Q and after a difference of t h another train with b km/h starts from Q towards station P, then both the trains will meet at a certain point after time T. Then,

$$T = \left(\frac{d \pm tb}{a+b}\right)$$

If second train starts after the first train, then t is taken as positive.

If seconds train starts before the first train, then t is taken as negative.

Example : 18 The distance between two stations P and Q is 110 km. A train with speed of 20 km/h leaves station P at 7:00 am towards station Q. Another train with speed of 25 km/h leaves station Q at 8:00 am towards station P. Then, at what time both trains meet?

Solution: Here, d = 110 km, t= 8:00 – 7:00 = 1 h, a= 20 km/h, b = 25 km/h

Time taken by trains to meet, $T = \left(\frac{d+tb}{a+b}\right) = \frac{110+(1)(25)}{20+25} = \frac{135}{45} = 3\ h$

They will meet at = 7:00 am + 3 h = 10:00 am

Technique : 7

The distance between two stations P and Q is d km. A train starts from P towards Q and another train starts from Q towards P at the same time and they meet at a certain point after t h. If train starting from P travels with a speed of x km/h slower or faster than another train, then

(i) Speed of faster train = $\left(\frac{d+tx}{2t}\right)$ km /h

(ii) Speed of slower train = $\left(\frac{d-tx}{2t}\right)$ km /h

Example: 19 The distance between two stations A and B is 138 km. A train starts from A towards B and another from B to A at the same time and they meet after 6 h. The train travelling from A to B is slower by 7 km/h compared to other train from B to A, then find the speed of the slower train?

Solution: Here, $d = 138\ km, t = 6\ h$ and $x = 7\ km/h$

Speed of slower train = $\frac{d-tx}{2t} = \frac{138-(6)(7)}{2(6)} = \frac{138-42}{12} = \frac{96}{12} = 8\ km/h$

Technique : 8

A train covers distance d between two stations P and Q in $t_1\ h$. If the speed of train is reduced by a km/h, then the same distance will be covered in $t_2\ h$.

(i) Distance between P and Q is d $= a\left(\frac{t_1 t_2}{t_2 - t_1}\right)\ km$

(ii) Speed of the train $= \left(\frac{at_2}{t_2 - t_1}\right)\ km/h.$

Example : 20 A train covers distance between two stations A and B in 2 h. If the speed of train is reduced by 6 km/h, then it travels the same distance in 3 h. Calculate the distance between two stations and speed of the train.

Solution : Here, $t_1 = 2\ h, t_2 = 3\ h, a = \frac{6km}{h}$ and d =?

(i) Distance between A and B is d $= a\left(\frac{t_1 t_2}{t_2 - t_1}\right)\ km = 6\left(\frac{2\times 3}{3-2}\right)$

$$d = 36\ km$$

(ii) Speed of the train $= \frac{at_2}{t_2 - t_1} = \frac{6\times 3}{3-2} = 18\ km/h.$

Technique: 9

Without stoppage, a train travels at an average speed of a and with stoppage, it covers the same distance at an average speed of b, then

Time of rest per hour $= \frac{Difference\ in\ average\ speeds}{Speed\ without\ stoppage} = \frac{a-b}{a}$ where, $a > b$.

Example: 21 Without stoppage, the speed of a train is 54 km/h and with stoppage, it is 45 km/h. For how many minutes, does the train stop per hour?

Solution: Here, a = 54 km/h and b = 45 km/h

According to the formula,

Required rest time $= \frac{a-b}{a} = \frac{54-45}{54} = \frac{9}{54} = \frac{1}{6}h = \frac{1}{6}\times 60 = 10\ min$

Technique :10

If two trains of equal lengths and different speeds take t_1 and t_2 time to cross a pole, then time taken by them to cross each other is

$$T = \frac{2t_1 t_2}{t_2 \pm t_1}$$

We use '+' sign, if trains are moving in opposite direction and '-' sign, if they are moving in same direction.

Example: 22 Two trains of equal length take 5 s and 6 s respectively to cross a pole. If these trains are moving in the same direction, then how long will they take to cross each other?

Solution: Here $t_1 = 5\ s$ and $t_2 = 6\ s$. According to the formula,

Required time $= \frac{2t_1 t_2}{t_2 - t_1} = \frac{2\times 5\times 6}{6-5} = 60\ s$