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DEPARTMENT OF AEROSPACE ENGINEERING

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Course	:	**19ASE310 FATIGUE AND FRACTURE MECHANICS**			

LECTURE NOTES

TOPIC: Effect of Mean Stress and Goodman, Gerber, and Soderberg Relations and Diagrams

Introduction

Fatigue failure is a common mode of failure in engineering components subjected to cyclic loading. Unlike static loading failures, fatigue occurs due to the repeated application of stress, often at stress levels lower than the material's ultimate tensile strength. The presence of **mean stress** in fatigue loading significantly influences the fatigue life of materials.

To account for the effect of mean stress, several empirical failure criteria have been developed, including the **Goodman, Gerber, and Soderberg relations**. These relationships help in estimating the fatigue strength of materials under different loading conditions, allowing engineers to design more reliable and durable components.

Definition of Mean Stress in Fatigue Analysis

Mean stress (σ_m) is the average stress in a loading cycle and is given by:

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

where:

- σ_m = Mean stress
- σ_{max} = Maximum applied stress
- σ_{min} = Minimum applied stress

The stress amplitude (σ_a) is given by:

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

The **stress ratio (R)**, which defines the cyclic loading condition, is given by:

$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

For fully reversed loading, **R = -1**, while for fluctuating loading, **R > 0**.

Effect of Mean Stress on Fatigue Life

Mean stress affects the fatigue strength of materials because it influences crack initiation and propagation. The key effects include:

- **Higher Mean Stress Reduces Fatigue Life**: When mean stress increases, the material experiences a higher overall stress range, leading to early crack initiation and reduced fatigue life.
- **Compressive Mean Stress Improves Fatigue Life**: If mean stress is compressive, it helps in closing cracks and delaying failure, thereby increasing fatigue resistance.
- **Ductile vs. Brittle Materials**: Ductile materials are more sensitive to mean stress, while brittle materials exhibit less dependence on mean stress in fatigue conditions.

To account for these effects, empirical models such as **Goodman, Gerber, and Soderberg relations** are used in fatigue design.

Fatigue Failure Criteria: Goodman, Gerber, and Soderberg Relations

These three failure criteria provide different approaches for estimating the fatigue strength of materials under non-zero mean stress conditions.

Goodman Relation

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The **Goodman relation** is a linear approximation that accounts for the combined effect of mean stress and alternating stress on fatigue failure. It is expressed as:

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$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = 1$$

where:

- σ_a = Alternating stress
- S_e = Endurance limit (fatigue limit)
- σ_m = Mean stress
- S_u = Ultimate tensile strength

The Goodman relation is **conservative**, making it widely used in fatigue design where safety is critical. It assumes that fatigue strength reduces linearly with increasing mean stress.

The **Gerber relation** is a more refined, **parabolic approximation** that provides a better estimate of fatigue failure in ductile materials. It is given by:

$$\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_u}\right)^2 = 1$$

This equation accounts for the nonlinear behavior of ductile materials and provides a **less conservative** estimate than the Goodman relation. However, it is not suitable for brittle materials, as they tend to fail in a more linear manner under cyclic loading.

Soderberg Relation

The **Soderberg relation** is the most **conservative** approach and is mainly used for highly reliable fatigue designs. It is given by:

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = 1$$

where S_y is the yield strength of the material.

Since yield strength is lower than ultimate strength, the Soderberg relation provides a **more restrictive design criterion**, making it suitable for critical applications where failure must be avoided at all costs.

Graphical Representation: Goodman, Gerber, and Soderberg Diagrams

These failure criteria can be visualized using **fatigue failure diagrams**, which help engineers determine the permissible stress levels for a given material under cyclic loading conditions.

Goodman Diagram

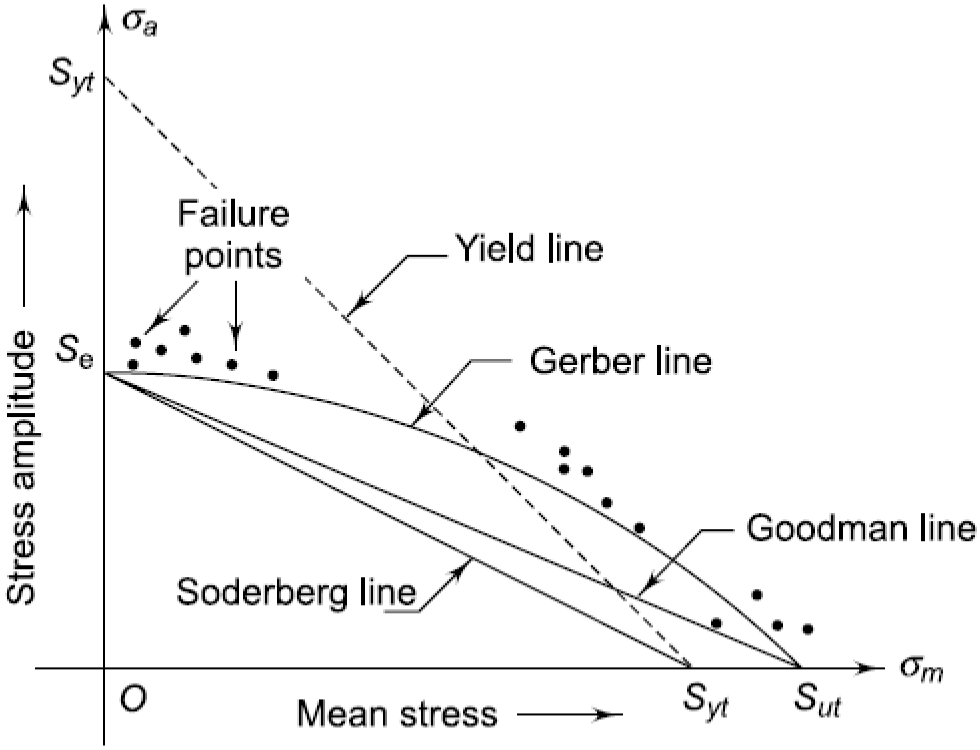
- A **straight-line approximation** connecting S_e on the **σ_a-axis** and S_u on the **σ_m-axis**.
- Conservative approach, widely used in engineering design.

Gerber Diagram

- A **parabolic curve** between S_e and S_u.
- Provides **more accurate fatigue predictions** for ductile materials.

Soderberg Diagram

- A **straight-line relation** between S_e and S_{γ}.
- Most **conservative**, ensuring high safety but often leading to overdesign.



Comparison of Goodman, Gerber, and Soderberg Relations

Criteria	Equation	Nature of Relationship	Conservatism	Suitability
Goodman	$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = 1$	Linear	Moderate	General engineering applications
Gerber	$\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_u}\right)^2 = 1$	Parabolic	Less conservative	Ductile materials
Soderberg	$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = 1$	Linear	Highly conservative	Safety-critical applications

Advantages of Using Fatigue Failure Criteria

- Helps in determining the **safe fatigue strength** of materials under fluctuating loads.
- Provides **accurate fatigue life predictions**, reducing failure risks in real-world applications.
- Aids in **optimizing material selection** and reducing overdesign, leading to cost savings.
- Allows engineers to design components with **increased reliability** and safety.

Limitations

- **Goodman Relation** is **overly conservative** for some materials.
- **Gerber Relation** is **not suitable for brittle materials**.
- **Soderberg Relation** may lead to **overly restrictive designs**, increasing material costs.
- None of these models account for **loading sequence effects** in real-world applications.

Applications of Mean Stress Corrections in Fatigue Design

- **Automotive Industry**: Fatigue life estimation of **engine components, connecting rods, and crankshafts**.
- **Aerospace Engineering**: Design of **aircraft fuselage, landing gears, and turbine blades**.
- **Structural Engineering**: Fatigue assessment of **bridges, offshore platforms, and railway tracks**.
- **Manufacturing**: Ensuring fatigue resistance in **machinery components and welded joints**.

The effect of mean stress is a crucial factor in fatigue analysis, influencing the durability of mechanical components. The **Goodman, Gerber, and Soderberg relations** provide engineers with valuable tools for estimating fatigue failure under varying stress conditions. While **Gerber's equation** is ideal for ductile materials, **Goodman's relation** is widely used for general applications, and **Soderberg's method** ensures high safety margins. By understanding these relations and applying them appropriately, engineers can design more **reliable, durable, and efficient** mechanical systems.