



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



Unit - II

Quantitative Ability - IV

Permutation & Combination

In our day-to-day life, we are interested to know the number of ways, in which a particular work can be done.

For this, we will have to know all the possible ways to do that work and it can be done with the help of Permutation and Combination.

Permutation ::

Each of the different arrangements which can be made by taking some or all of a given number of things or objects at a time, is called a Permutation. Permutation implies arrangement where order of the things is important.

For example, The Permutations of three times a, b and c taken two at a time are ab, ba, ac, ca, cb and bc. Since, the order in which the items are taken, is important, ab and ba are counted as two different Permutation.

Let r and n be positive integers such that $1 \leq r \leq n$. Then, the number of Permutations of n different things, taken r at a time, is denoted by ${}^n P_r$ (or) $P(n, r)$.

Formula for Permutation,
$${}^n P_r = \frac{n!}{(n-r)!}$$



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Cases of Permutation :

There are several cases of Permutation

1. Formation of numbers with given digits :

In these type of questions, it is asked to form numbers with some different digit. These digit can be used with repetition or without repetitions.

Example: 1 How many numbers of four digits can be formed with the digits 1, 2, 3, 4 and 5? (Repetition of digits is not allowed.)

Solution: There are five numbers and number of places to be filled up = 4

So, required number of numbers is

$${}^5P_4 = \frac{5!}{(5-4)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{1} = 120$$

Example: 2 How many numbers between 400 and 1000 can be made with the digits 2, 3, 4, 5, 6 and 0?

Solution: Here, nothing is mentioned about repetition of digits, therefore we will assume that repetition of digits is not allowed.

Now, any number of three digits to be in between 400 and 1000 its hundred place must be occupied by 4 or 5 or 6 (because if it will start with 0, 2 and 3, then it will not lie between 400 to 1000).

So, remaining two places can be filled up by five digits (since six digits 2, 3, 4, 5, 6, 0 are given and if the first place is occupied by 4, then remaining two can be filled up by five digits 2, 3, 5, 6 and 0).



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So, required number of ways to fill first place = 3 and required number of ways to fill remaining two places

$$= {}^5P_2$$

$$\begin{aligned} \text{Required number of numbers} &= 3 \times {}^5P_2 \\ &= \frac{3 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 60 \end{aligned}$$

2. Formation of words with given letters :-

These questions are very much similar to previous case questions but here in place of numbers, words are formed from a set of English alphabets given in the form of a word.

Note:- Number of Permutations of n objects out of which p are alike and are of one type, q are alike and are of second type and r are alike and are of third type $\frac{n!}{p!q!r!}$

Example: 3 In how many ways, can the letters of the word 'DIRECTOR' be arranged, so that the three vowels are never together?

Solution:- Total number of letters = 8 and total number of vowels = 3.

Here, R occurs two times.

$\therefore$ Total number of arrangements where there is no restriction = $\frac{8!}{2!}$

= 20160, but when

three vowels are together, regarding them as one letter, we have only $5+1=6$ letters.

These 6 letters can be arranged in $\frac{6!}{2!}$ ways, since R occurs twice.



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Number of Permutations of n different objects taken r at a time, when repetition is allowed $= n^r$.

Example: 5 In how many ways, 3 books can be given away to 7 boys, where each boy is eligible for any of the books?

Solution: Each of the three books can be given away to anyone of the 7 boys in 3 ways.

Required number of ways $= 7^3 = 343$.

Example: 6 A gentleman has 6 friends to invite. In how many ways, can he send invitation cards to them, if he has three servants to carry the cards?

Solution: Invitation cards may be sent to each of the six friends by anyone of the three servants in 3 ways.

Required ways $= 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6 = 729$

Some other examples of Permutation:

Example: 7 If $n P_4 = 360$, find n .

Solution: Given, $n P_4 = 360$

$$\left[\because n P_r = \frac{n!}{(n-r)!} \right]$$

$$\frac{n!}{(n-4)!} = 360$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 360$$

$$n(n-1)(n-2)(n-3) = 360 = 6 \times 5 \times 4 \times 3$$

$$n = 6$$



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Also, three vowels can be arranged among themselves in $3!$ ways.

Hence, number of arrangements when the three vowels are together.

$$= 3! \times \frac{6!}{2!}$$

$$= 2160$$

Number of arrangements, so that the three vowels are never together

$$= 20160 - 2160$$

$$= 18000$$

3. Arrangement of Persons in a row or at a round table :

These type of question are based on arrangement of Person (boy or girls etc) in a straight line facing some direction or around some circular object like table etc.

Note: Number of Permutations of n objects taken all at a time is $n!$, when repetition is not allowed.

Example: 4 In how many different ways 5 girls can be seated in a row?

Solution: Number of ways in which 5 girls can be seated in a

$$\text{row} = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

4. Arrangement of books on a Shelf, etc.

In such questions arrangement of books is done into a shelf in a row or one over the other.

Note: Questions based on sending invitation to different Persons are similar to questions based on arrangement of books.



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Example: 8 If ${}^{10}P_r = 720$, find r .

Solution: Given, ${}^{10}P_r = 720$

$$\frac{10!}{(10-r)!} = 720 = 10 \times 9 \times 8$$

$$\frac{10!}{(10-r)!} = \frac{10 \times 9 \times 8 \times 7!}{7!} = \frac{10!}{7!}$$

$$(10-r)! = 7!$$

$$10-r = 7$$

$$-r = 7-10$$

$$-r = -3$$

$$r = 3$$

Combination :

Combination of things means selection of things.

Here, order of things has no importance.

For example, The Combination of two letters from the group of three letters A, B and C would be as follows AB, BC, AC.

Here, we make groups. So, AB and BA as a group is same.

Obviously, if order matters, then AB and BA are not same.

Formula for Combination, ${}^nC_r = \frac{n!}{r!(n-r)!}$

It signifies number of groups formed from n different things,

When r things are taken into consideration.

Note: ${}^nC_n = {}^nC_0 = 1$; ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$;



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$$nC_r = nC_{n-r} \quad \text{or} \quad nC_r = \frac{(nPr)}{r!}$$

$$nC_0 + nC_1 + nC_2 + \dots + nC_n = 2^n$$

$$nC_1 + nC_2 + nC_3 + \dots + nC_n = 2^n - 1$$

Cases of Combination :

There are Several cases of combination

1. Formation of Committee from a given set of Persons.

These questions are based on formation of a Committee consisting of some members (male / female) from a group of Persons following a certain condition.

Example : 9 In how many ways can 5 members form a committee out of 10 be Selected so that

(i) two Particular members must be included.

(ii) two Particular members must not be included.

Solution : (i) When two Particular members are included then, we have to select $5 - 2 = 3$ members out of $10 - 2 = 8$.

The required number of ways = $C(8, 3) = \frac{8!}{3!5!} = \frac{8 \times 7 \times 6}{6} = 56$.

(ii) When two Particular members are not included then, we have to select 5 members out of $10 - 2 = 8$.

The required numbers of ways = $C(8, 5) = \frac{8!}{5!3!} = \frac{8 \times 7 \times 6}{6} = 56$.

2. Selection of questions from question Paper etc.

In such question, a question Paper is given with one or more Parts and the different ways in which some specified number of



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questions can be attempted is asked.

Example: 10 A question paper has two parts, Part A and Part B each containing 10 questions. If the student has to choose 8 from Part A and 5 from part B, in how many ways can he choose the question?

Solution: The required number of ways

$$= {}^{10}C_8 \cdot {}^{10}C_5$$

$$= \frac{10!}{8!2!} \times \frac{10!}{5!5!} = \frac{10 \times 9}{2} \times \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2}$$

$$= 5 \times 9 \times 3 \times 2 \times 7 \times 6$$

$$= 11340$$

Some other examples of Combination:

Example: 11 Find the value of the following:

i) ${}^{15}C_{11}$

Solution: ${}^{15}C_{11} = \frac{15!}{11!(15-11)!} = \frac{15 \times 14 \times 13 \times 12 \times 11!}{11! \times 4!}$

$$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2}$$

$$= 1365$$

$${}^nC_r = \frac{n!}{(n-r)!r!}$$

ii) ${}^{10}C_4$

Solution: ${}^{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{4! \times 6!}$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$



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Example: 12 Find the number of ways, in which 7 books can be selected out of 10 books available.

Solution: Total number of books = 10

Number of books to be selected = 7

$$\text{Required number of ways} = {}^{10}C_7 = \frac{10!}{7!(10-7)!}$$

$$= \frac{10 \times 9 \times 8 \times 7!}{7! \times 3 \times 2 \times 1}$$

$$= 120$$

Factorial :-

Factorial of a number can be defined as the product of all natural numbers upto that number.

$$\text{i.e. } n! = n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times \dots \times 1 = n \times (n-1)!$$

$$4! = 4 \times 3 \times 2 \times 1 = 4 \times 3!$$

$$11! = 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1.$$

Note: Factorial of negative number and integers is not defined.

$${}^nP_n = n! , {}^nP_0 = 1.$$

Fundamental Principles of Counting :-

Multiplication Principle :-

If an operation can be performed in m different ways, following which a second operation can be performed in n different ways, then the two operations in succession can be performed in $m \times n$ ways.