Example:12 Find the number of ways, in which 7 books can be selected out of 10 books available.

Solution:- Total number of books = 10

Number of books to be selected = 7

Required number of ways $= {}^{10}C_7 = \frac{10!}{7!(10-7)!}$

$= \frac{10 \times 9 \times 8 \times 7!}{7! \times 3 \times 2 \times 1}$

$= 120$

Factorial :-

Factorial of a number can be defined as the Product of all natural numbers upto that number.

ie.) $n! = n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times \cdots \times 1 = n \times (n-1)!$

$4! = 4 \times 3 \times 2 \times 1 = 4 \times 3!$

$11! = 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1.$

Note :- Factorial of negative number and integers is not defined.

$nP_n = n!$, $nP_0 = 1$.

Fundamental Principles of Counting :-

Multiplication Principle :-

If an operation can be performed in m different ways, following which a second operation can be performed in n different ways, then the two operations in succession can be performed in $m \times n$ ways.

This can be extended to any finite number of mutually exclusive operations.

In general, if there are n jobs to Perform and each can be Performed in m_i ways $(i = 1, 2, \ldots n)$ then number of ways of doing all things simultaneously is $m_1 \times m_2 \times m_3 \times \ldots \times m_n$.

Here, the jobs Performed are mutually inclusive.

Example: 13 A hall has 12 gates. In how many ways, can a man enter the hall through one gate and come out through a different gate?

Sol:- Since, there are 12 ways of entering into the hall, the man come out through a different gate in 11 ways.

Hence, by the fundamental Principle of multiplication,

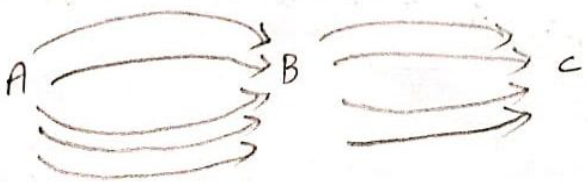
Total number of ways is $12 \times 11 = 132$.

Example:-14 There are three stations A, B and C and five routes for going from station A to B and four routes from station B to C. Find the number of different ways through which a person can go from A to C via B.

Solu:- Since, there are five routes for going from A to B and four routes from B to C.

So, number of different ways to go from A to C via B

$= 5 \times 4 = 20$.



Addition Principle:.

If an operation can be Performed in m different ways and another operation, which is independent of the first operation, can be Performed in n different ways, then either of the two operations can be Performed in (m+n) ways. This can be extended to any finite number of mutually exclusive operations.

In general, if there are n independent jobs, each of which can be Performed in m_i ways, then the total number of ways of performing all things simultaneously is $m_1 + m_2 + \cdots + m_n$.

Here, the jobs Performed are mutually exclusive.

Example:15 There are 25 students in a class with 15 boys and 10 girls. The class teacher selects either a boy or girl for monitor Post of the class. In how many ways, the class teacher can make this selection?

Sol:. There are 15 boys and 10 girls and monitor selected can be anyone from the given students.

Required number of ways = 15 + 10 = 25.

Fast Track Formulae to Solve the Questions:.

Formula: 1

If $nC_x = nC_y$, then $x = y$ or $x + y = n$

Example:16 If $15C_{3r} = 15C_{r+3}$, find r.

Sol:. $15C_{3r} = 15C_{r+3}$

$3r = r+3$ (or) $3r + r + 3 = 15 \Rightarrow r = \frac{3}{2}$ (or) $r = 3$.

r Cannot be fraction, $r = 3$.

Formula : 2

Number of circular Permutations of n different objects = $(n-1)!$

Example : 17 Find the number of ways, in which 10 boys can form a ring?

Sol:. Total number of boys = 10

According to formula,

Total number of ways in which ring can be formed = $(10-1)!$

$= 9!$

$= 362880$

Formula : 3

In a circular Permutation, if clockwise and anti-clockwise arrangements are considered to be same, then

Number of circular Permutations of n objects = $\frac{(n-1)!}{2}$

Example: 18 Find the total number of ways, in which 10 beads can be strung into a necklace?

Sol:. Total number of beads = 10

According to the formula,

Required numbes of ways = $\frac{(10-1)!}{2} = \frac{9!}{2} = \frac{362880}{2}$

$= 181440$

Formula : 4

Number of ways to declare the result where 'n' match are Played $= 2^n$.

Example: 19 In a cricket tournament 5 matches were played, then in how many ways a result can be declared?

Sol: Total ways to declare the result $= 2^n = 2^5 = 32$ ways.

Formula: 5

Let there are n Persons in a hall. If every Person shakes his hand with every other Person only once, then total number of handshakes $= {}^nC_2 = \frac{n(n-1)}{2}$.

Note: If in place of handshakes each person gives a gift to another person, then formula changes to $= n(n-1)$.

Number of diagonals in a polygon of n sides $= {}^nC_2 - n$.

Example: 20 In a Party, every Person shakes his hand with every other Person only once. If total number of handshakes is 210, then find the number of Persons.

Sol: Let number of Persons be n.

According to question, ${}^nC_2 = 210$

$$\frac{n(n-1)}{2} = 210$$

$$n(n-1) = 420$$

$$= 21 \times 20$$

$$n = 21$$

Formula: 6

If there are n non-collinear points in a plane, then

i) Number of straight lines formed $= {}^nC_2$

ii) Number of triangles formed $= {}^nC_3$

iii) Number of quadrilaterals formed $= {}^nC_4$

Example: 21 In a plane, there are 16 non-collinear Points. Find the number of straight lines formed.

Sol: $n = 16$

Required number of straight lines formed $= {}^nC_2$

$= {}^{16}C_2$

$= \frac{16!}{2!(16-2)!}$

$= \frac{8 \times 15 \times 14!}{2 \times 14!}$

$= 8 \times 15$

$= 120$

Formula: 7

If there are n points in a plane out of which m are collinear, then i) Number of straight lines formed $= {}^nC_2 - {}^mC_2 + 1$

ii) Number of triangles formed $= {}^nC_3 - {}^mC_3$

Example: 22 In a plane, there are 11 Points, out of which 5 are collinear. Find the number of ~~things~~ triangles made by these points.

Sol: $n = 11, m = 5$

Required number of triangles $= {}^nC_3 - {}^mC_3 = {}^{11}C_3 - {}^5C_3$

$= \frac{11 \times 10 \times 9}{3 \times 2 \times 1} - \frac{5 \times 4 \times 3}{3 \times 2 \times 1}$

$= 165 - 10 = 155$.