



DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

NEWTON'S METHOD (or) NEWTON'S RAPHSOON METHOD

Formula : $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, provided $f'(x_n) \neq 0$

① Find the smallest positive root of the eqn. $x^3 - 2x + 0.5 = 0$.

Let $f(x) = x^3 - 2x + 0.5$; $f'(x) = 3x^2 - 2$

Now $f(0) = 0.5$ (+ve)

$f(1) = -0.5$ (-ve)

∴ The root lies btwn. 0 & 1.

Since $|f(0)| = |f(1)|$, let us assume $x_0 = 0$

Newton's Raphsoon formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

putting $n=0$, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$= 0 - \frac{0.5}{-2} = 0.25$$

putting $n=1$, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.25 - \frac{f(0.25)}{f'(0.25)} = 0.2586$

putting $n=2$, $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.2586 - \frac{f(0.2586)}{f'(0.2586)} = 0.2586$

Since x_2 & x_3 are equal root, The smallest positive root is 0.2586



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② Compute the real root of $x \log_{10} x = 1.2$ correct to three decimal places using Newton's-Raphson Method

Let $f(x) = x \log_{10} x - 1.2$; $f'(x) = \log_{10} x + 1$

$x \log_{10} x = 1.2$
 $x \log_{10} x = 1.2$
 0.4243
 $f(0) = -1.2$ (-ve) $f(1) = 0.4343$ $x \log_{10} x - 1.2$
 $f(1) = -1.2$ (-ve) $f(10) = 0.4343 \log_{10} 10 + 0.4343$

$f(2) = -0.5980$ (-ve)

$f(3) = 0.2313$ (+ve)

∴ The root lies between 2 & 3.

Since $|f(2)| > |f(3)|$, let us assume $x_0 = 3$.

[Since 0.2313 is nearer to 0 than 0.5980, let us assume the root '3' as x_0 .]

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.8436$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.7812$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7567$$



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$$x_4 = 2.7469$$

$$x_5 = 2.7431$$

$$x_6 = 2.7416$$

$$x_7 = 2.7410$$

$$x_8 = 2.7407$$

$$x_9 = 2.7406$$

$$x_{10} = 2.7406$$

$$\begin{array}{r} 0.2313 \\ 3 - \overline{) 1.4771} \end{array}$$

Since x_9 & x_{10} are equal, therefore the required root is 2.7406.

③ Find the positive root of $2x^3 - 3x - 6 = 0$

$$\text{let } f(x) = 2x^3 - 3x - 6 ; f'(x) = 6x^2 - 3$$

$$f(0) = -6 \quad (-ve)$$

$$f(1) = -7 \quad (-ve)$$

$$f(2) = 4 \quad (+ve)$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$



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$$x_1 = 1.80952$$

$$x_2 = 1.78419$$

$$x_3 = 1.78377$$

$$x_4 = 1.78377$$

Since x_3 & x_4 are equal, Therefore the required root is 1.78377



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1) Find the -ve root of $x^3 - \sin x + 1 = 0$. Radian -1

Let $f(x) = x^3 - \sin x + 1$; $f'(x) = 3x^2 - \cos x$.
 $\sin(-x) = -\sin x$

$$f(-x) = -x^3 + \sin x + 1$$

$$f(0) = 1 \quad (+ve)$$

$$f(1) = -1 + \sin 1 + 1 = 0.8414 \quad (+ve)$$

$$f(-2) = -8 + \sin 2 + 1 = -6.0106 \quad (-ve)$$

$\therefore$ The root lies between -1 & -2.

Since $|f(-1)| < |f(-2)|$, let us assume $x_0 = -1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1.3421 \quad \begin{matrix} -1.3668 & -1.5095 \end{matrix}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -1.2564 \quad \begin{matrix} -1.0346 & -1.2967 \\ -1.2571 \end{matrix}$$



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$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -1.2491$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -1.2490$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = -1.2490$$

Since x_4 & x_5 are equal, therefore the required root is -1.2490 .

14. (5) Find the root of $x \tan x = 1.28$. Soln: 0.9382 , the requ. root

(6) By NRM find a non-zero root of $x^2 + 4 \sin x = 0$.

Soln: The roots are -1 & -2 . The requ. root is -1.9338 .