Gauss Elimination Method:

(1) Solve the system of equations by Gaussian elimination method.

$$10x - 2y + 3z = 23$$
$$2x + 10y - 5z = -33$$
$$3x - 4y + 10z = 41$$

The given system is equivalent to $AX = B$

(ii)
$$\begin{pmatrix} 10 & -2 & 3 \\ 2 & 10 & -5 \\ 3 & -4 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 23 \\ -33 \\ 41 \end{pmatrix}$$

Now $[A, B] = \begin{bmatrix} 10 & -2 & 3 & 23 \\ 2 & 10 & -5 & -33 \\ 3 & -4 & 10 & 41 \end{bmatrix}$

Let us reduce augmented matrix $[A, B]$ to upper triangular matrix.

Step 1: Fix the first row, change 2 & 3 row with row 1

$$[A,B] \sim \begin{bmatrix} 10 & -2 & 3 & 23 \\ 0 & 10.4 & -5.6 & -37.6 \\ 0 & -3.4 & 9.1 & 34.1 \end{bmatrix} \begin{matrix} \\ R_2 \leftrightarrow R_2 - \frac{2}{10} R_1 \\ R_3 \leftrightarrow R_3 - \frac{3}{10} R_1 \end{matrix}$$

Step 2: Fix 1 & 2 row, change 3 row with 2nd row

$$\sim \begin{bmatrix} 10 & -2 & 3 & 23 \\ 0 & 10.4 & -5.6 & -37.6 \\ 0 & 0 & 7.26 & 21.80 \end{bmatrix} R_3 \leftrightarrow R_3 - \left(\frac{-3.4}{10.4}\right) R_2$$

which is an upper triangular matrix.

Step 3: Back Substitution.

We get, $7.26z = 21.80 \Rightarrow z = 3$

$10.4y - 5.6z = -37.6 \Rightarrow y = -2$

$10x - 2y + 3z = 23 \Rightarrow x = 1$

Hence soln. is $x = 1, y = -2, z = 3$

checking: $10x - 2y + 3z = 23$
$10(1) - 2(-2) + 9 = 23$