



DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

ITERATIVE METHODS (or) INDIRECT METHODS :

GAUSS JACOBI (or) JACOBI'S METHOD :

Let the system of simultaneous equations be

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

Assume :
 $|a_1| > |b_1| + |c_1|$
 $|b_2| > |a_2| + |c_2|$
 $|c_3| > |a_3| + |b_3|$
 (The diagonal elts. should dominant, so that, the iteration process can be applied)
 This system of equations can also be written as:

$$x = \frac{1}{a_1} (d_1 - b_1y - c_1z)$$

$$y = \frac{1}{b_2} (d_2 - a_2x - c_2z)$$

$$z = \frac{1}{c_3} (d_3 - a_3x - b_3y)$$

Let the first approximation be x_0, y_0 and z_0 .
 Sub. x_0, y_0 , and z_0 in (2), we get.

$$x_1 = \frac{1}{a_1} (d_1 - b_1y_0 - c_1z_0)$$

$$y_1 = \frac{1}{b_2} (d_2 - a_2x_0 - c_2z_0)$$

$$z_1 = \frac{1}{c_3} (d_3 - a_3x_0 - b_3y_0)$$

Sub. the values of x_1, y_1, z_1 in (2), we get



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Coimbatore – 35



DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

$$x_2 = \frac{1}{a_1} (d_1 - b_1 y_1 - c_1 z_1)$$

$$y_2 = \frac{1}{b_2} (d_2 - a_2 x_1 - c_2 z_1)$$

$$z_2 = \frac{1}{c_3} (d_3 - a_3 x_1 - b_3 y_1)$$

This process is repeated till the difference btwn. two consecutive approximations is negligible.

1) Solve the following system by Gauss-Jacobi method
[Form the Table]

$$10x - 5y - 2z = 3$$

$$4x - 10y + 3z = -3$$

$$x + 6y + 10z = -3$$

Soln:

$$a_1: |10| > |-5| + |-2|$$

$$b_2: |-10| > |4| + |3|$$

$$c_3: |10| > |1| + |6|$$

Since the diagonal elts are dominant, the iteration process is applied here.



DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

The gn. system can be written as,

$$x = \frac{1}{10} (3 + 5y + 2z)$$

$$y = \frac{1}{10} (3 + 4x + 3z)$$

$$z = \frac{1}{10} (-3 - x - 6y)$$

I iteration: let the initial values be

$$x_1 = \frac{1}{10} (3 + 5y_0 + 2z_0)$$

$$y_1 = \frac{1}{10} (3 + 4x_0 + 3z_0)$$

$$z_1 = \frac{1}{10} (-3 - x_0 - 6y_0)$$

let the initial values be $x_0 = y_0 = z_0 = 0$

$$x_1 = \frac{1}{10} [3 + 5(0) + 2(0)] = 0.3$$

$$y_1 = \frac{1}{10} [3 + 4(0) + 3(0)] = 0.3$$

$$z_1 = \frac{1}{10} [-3 - 0 - 6(0)] = -0.3$$

II iteration:

$$x_2 = \frac{1}{10} (3 + 5y_1 + 2z_1)$$

$$y_2 = \frac{1}{10} (3 + 4x_1 + 3z_1)$$

$$z_2 = \frac{1}{10} (-3 - x_1 - 6y_1)$$



DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

$$x_2 = \frac{1}{10} [3 + 5(0.3) + 2(-0.3)] = 0.39$$

$$y_2 = \frac{1}{10} [3 + 4(0.3) + 3(-0.3)] = 0.33$$

$$z_2 = \frac{1}{10} [-3 - 1(0.3) - 6(0.3)] = -0.51$$

III iteration:

$$x_3 = 0.363$$

$$y_3 = 0.303$$

$$z_3 = -0.537$$

IV iteration:

$$x_4 = 0.3441$$

$$y_4 = 0.2841$$

$$z_4 = -0.5181$$

V iteration:

$$x_5 = 0.3384$$

$$y_5 = 0.2822$$

$$z_5 = -0.5048$$

VI iteration:

$$x_6 = 0.3401$$

$$y_6 = 0.2839$$

$$z_6 = -0.5031$$

VII iteration:

$$x_7 = 0.3413$$

$$y_7 = 0.2851$$

$$z_7 = -0.5043$$

VIII iteration:

$$x_8 = 0.3416$$

$$y_8 = 0.2852$$

$$z_8 = -0.50519$$

IX iteration:

$$x_9 = 0.3415$$

$$y_9 = 0.28511$$

$$z_9 = -0.5052$$

X iteration:

$$x_{10} = 0.34148$$

$$y_{10} = 0.28504$$

$$z_{10} = -0.50522$$



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DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

From 2 & 3 iteration, approximately we get

$$x \approx 0.3415$$

$$y \approx 0.2850$$

$$z = -0.5052$$

② Solve the following equations using Jacobi's iteration method:

$$30x - 2y + 3z = 75$$

$$x + 17y - 2z = 48$$

$$x + y + 9z = 15$$

Suppose: $x + 17y - 2z = 48$

$$x = \frac{48 - 17y + 2z}{1}$$

$$30x - 2y + 3z = 75$$

Now put $x = \frac{48 - 17y + 2z}{1}$