NEWTON'S METHOD (or) NEWTON'S RAPHSON METHOD

Formula: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, provided $f'(x_n) \neq 0$

(1) Find the smallest positive root of the eqn. $x^3 - 2x + 0.5 = 0$.

Let $f(x) = x^3 - 2x + 0.5$; $f'(x) = 3x^2 - 2$

Now $f(0) = 0.5$ (+ve)

$f(1) = -0.5$ (-ve)

$\therefore$ The root lies btwn. 0 & 1.

Since $|f(0)| = |f(1)|$, let us assume $x_0 = 0$

Newton's Raphson formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

putting $n=0$, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$= 0 - \frac{0.5}{-2} = 0.25$$

putting $n=1$, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.25 - \frac{f(0.25)}{f'(0.25)} = 0.2586$

putting $n=2$, $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.2586 - \frac{f(0.2586)}{f'(0.2586)} = 0.2586$

Since x_2 & x_3 are equal root, the smallest positive root is 0.2586

(2) Compute the real root of $x \log_{10} x = 1.2$ correct to three decimal places using Newton's - Raphson Method

Let $f(x) = x \log_{10} x - 1.2$; $f'(x) = \log_{10} x + 1$

$f(0) = -1.2$ (-ve) $\quad f(x) = 0.4343\, x \log_e x - 1.2$

$f(1) = -1.2$ (-ve) $\quad f'(x) = 0.4343 \log_e x + 0.4343$

$f(2) = -0.5980$ (-ve)

$f(3) = 0.2313$ (+ve)

∴ The root lies between 2 & 3.

Since $|f(2)| > |f(3)|$, let us assume $x_0 = 3$

[since 0.2313 is nearer to '0' than 0.5980, we assume the root '3' as x_0.]

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.8436$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.7812$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7567$$

$x_4 = 2.7469$

$x_5 = 2.7431$

$x_6 = 2.7416$

$x_7 = 2.7410$

$x_8 = 2.7407$

$x_9 = 2.7406$

$x_{10} = 2.7406$

$3 - \frac{0.2313}{1.4771}$

Since x_9 & x_{10} are equal, therefore the required root is 2.7406.

(3) Find the positive root of $2x^3 - 3x - 6 = 0$

Let $f(x) = 2x^3 - 3x - 6$; $f'(x) = 6x^2 - 3$

$f(0) = -6$ (-ve)

$f(1) = -7$ (-ve)

$f(2) = 4$ (+ve)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_1 = 1.80952$

$x_2 = 1.78419$

$x_3 = 1.78377$

$x_4 = 1.78377$

Since x_3 & x_4 are equal, Therefore the required root is 1.78377

1) Find the -ve root of $x^3 - \sin x + 1 = 0$. Radian

Let $f(x) = x^3 - \sin x + 1$; $f'(x) = 3x^2 - \cos x$.

$\sin(-x) = -\sin x$.

$f(-x) = -x^3 + \sin x + 1$

$f(-0) = 1 \quad (+ve)$

$f(-1) = -1 + \sin 1 + 1 = 0.8414 \quad (+ve)$

$f(-2) = -8 + \sin 8 + 1 = -6.0106 \quad (-ve)$

$\therefore$ The root lies btwn -1 & -2.

Since $|f(-1)| < |f(-2)|$, let us assume $x_0 = -1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1.3421$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -1.2564$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -1.2491$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -1.2490$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = -1.2490$$

Since x_4 & x_5 are equal, therefore the required root is -1.2490.

H.W. (5) Find the root of $x \tan x = 1.28$. Soln: 0.9382, the requ. root

(6) By NRM find a non-zero root of $x^2 + 4\sin x = 0$.
Soln: The roots are -1 & -2. The requ. root is -1.9338.