(1) Obtain Newton's iterative formula for finding $\sqrt{N}$ where N is a +ve real no. Hence evaluate $\sqrt{5}$.

Soln: Let $x = \sqrt{N}$

$$x^2 = N$$

$$\Rightarrow x^2 - N = 0.$$

$$f(x) = x^2 - N \,;\quad f'(x) = 2x.$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \left(\frac{x_n^2 - N}{2x_n}\right)$$

$$= \frac{2x_n^2 - x_n^2 + N}{2x_n}$$

$$= \frac{x_n^2 + N}{2x_n}$$, is an iterative formula for $\sqrt{N}$.

To find $\sqrt{5}$.

$$x = \sqrt{5}$$

$$x^2 - 5 = 0$$

$$\Rightarrow f(x) = x^2 - 5 \;;\quad f'(x) = 2x$$

$f(0) = -5 \quad (-ve)$
$f(1) = -4 \quad (-ve)$
$f(2) = -1 \quad (-ve)$
$f(3) = 4 \quad (+ve)$

$\therefore$ The root lies btwn. 2 & 3.

Since $|f(2)| < |f(3)|$, let us assume $x_0 = 2$, (since the value 1 is nearer to '0' than 4)

$$\left\{ \begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 2 - \frac{(-1)}{4} = \frac{9}{4} = 2.25 \end{aligned} \right]$$

Here $x_{n+1} = \dfrac{x_n^2 + N}{2x_n}$

$$x_1 = \frac{x_0^2 + N}{2x_0}$$

Here $N = 5$, $x_0 = 2$.

$$\Rightarrow x_1 = \frac{2^2 + 5}{2(2)} = \frac{9}{4} = 2.25$$

$$x_2 = \frac{x_1^2 + N}{2x_1}$$

$$= \frac{(2.25)^2 + 5}{2(2.25)} = 2.2361$$

$$x_3 = \frac{x_2^2 + N}{2x_2}$$

$$= \frac{(2.2361)^2 + 5}{2(2.2361)} = 2.2360$$

$$x_4 = 2.2360$$

Since x_3 & x_4 are equal, the required root is 2.2360

HW (2) Find the value of $\sqrt{142}$. Soln: The requ. root is 11.9164

(3) Find the value of $\sqrt{35}$ Soln: The requ. root is 5.9160

(1) Find the iterative formula for finding the value of $\frac{1}{N}$ (reciprocal of N) where N is a real no., using NRM. Hence evaluate $\frac{1}{26}$ correct to 4 decimal places.

Soln: Let $x = \frac{1}{N}$

(i.e) $N = \frac{1}{x}$.

Let $f(x) = \frac{1}{x} - N$; $f'(x) = -\frac{1}{x^2}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \left(\frac{\frac{1}{x_n} - N}{-\frac{1}{x_n^2}}\right)$$

$$= x_n + x_n^{\cancel{2}} \left(\frac{1 - Nx_n}{\cancel{x_n}}\right)$$

$= x_n + x_n - N x_n^2$

$= 2x_n - N x_n^2$, is the Iterative formula.

To find $\frac{1}{26}$; $N = 26$.

$f(x) = \frac{1}{x} - 26$; $f'(x) = -\frac{1}{x^2}$.

$f(0) = -26$ (-ve)

$f(1) = -25$ (-ve)

$f(2) = -25.5$ (-ve) [It's impossible to find the roots]

Let us take $x_0 = \frac{1}{25} = 0.04$, nearer to the given N.

$x_0 = 0.04$.

WKT, $x_{n+1} = 2x_n - N x_n^2$.

$x_1 = 2x_0 - 26 x_0^2$

$= 2(0.04) - 26(0.04)^2$

$= 0.0384$

$x_2 = 0.0384$.

Since x_1 & x_2 are equal, the value of $\frac{1}{26} = 0.0384$

H/W (2) Find the value of $\frac{1}{19}$ Soln: 0.0526

(3) Derive Newton's algorithm for finding the p^{th} root of a number N. & find the value of $(24)^{1/3}$

Soln:

Let $x = N^{1/p}$.

$x^p = N$.

$\Rightarrow x^p - N = 0$.

Let $f(x) = x^p - N$. ; $f'(x) = px^{p-1}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \left(\frac{x_n^p - N}{px_n^{p-1}}\right)$$

$$= \frac{px_n^p - x_n^p + N}{px_n^{p-1}} = \frac{(p-1)x_n^p + N}{px_n^{p-1}}$$

To find $(24)^{1/3}$:

Here $N = 24$, $p = 3$.

$f(x) = x^p - N$.

$f(x) = x^3 - 24$

$f(x) = x^3 - 24$

$f(0) = -24$ (-ve)

$f(1) = -23$ (-ve)

$f(2) = -16$ (-ve)

$f(3) = 3$ (+ve), The root lies btwn. 2 & 3

Since $|f(2)| > |f(3)|$, let us assume $x_0 = 3$.

$$x_{n+1} = \frac{(3-1)x_n^3 + 24}{3x_n^{3-1}} = \frac{2x_n^3 + 24}{3x_n^2}$$

$$x_1 = \frac{2x_0^3 + 24}{3x_0^2} = 2.8888$$

$x_2 = 2.8845$

$x_3 = 2.8844$

$x_4 = 2.8844$

Since $x_3 = x_4$, The required root is 2.8844.