SOLUTION OF LINEAR SYSTEM

There are two types of methods to solve linear algebraic equations

(i) Direct Method:

(a) Gauss Elimination Method

(b) Gauss Jordan Method

(ii) Indirect Method (or) Iterative Method:

(a) Gauss Jacobi Method

(b) Gauss seidel Method

Gauss Elimination Method:

Let us consider the 'n' linear equations

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$\cdots\cdots\cdots\cdots$$

$$\cdots\cdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

where a_{ij} and b_i are known constants and x_i's are unknowns

The above eqn. is equivalent to $AX = B$

where $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$ $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ and $B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$

Now our aim is to reduce the augmented matrix $[A, B]$ to upper triangular matrix.

Augmented matrix is

$$[A, B] = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \cdot & & & & \\ a_{n1} & a_{n2} & \cdot & a_{nn} & b_n \end{pmatrix}$$

which is reduced to upper triangular matrix, as,

$$\begin{pmatrix} a_{11} & a_{12} & \cdot & a_{1n} & b_1 \\ 0 & b_{22} & \cdot & b_{2n} & c_2 \\ \vdots & \vdots & & & \\ 0 & 0 & & & d_n \end{pmatrix}$$

By back substitution method we get the values for $x_n, x_{n-1}, \ldots x_2, x_1$.

① Solve the system of equations by Gaussian elimination method.

$$10x - 2y + 3z = 23$$
$$2x + 10y - 5z = -33$$
$$3x - 4y + 10z = 41$$

The given system is equivalent to $AX = B$

(ie)
$$\begin{pmatrix} 10 & -2 & 3 \\ 2 & 10 & -5 \\ 3 & -4 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 23 \\ -33 \\ 41 \end{pmatrix}$$

Now
$$[A, B] = \begin{bmatrix} 10 & -2 & 3 & 23 \\ 2 & 10 & -5 & -33 \\ 3 & -4 & 10 & 41 \end{bmatrix}$$

Let us reduce augmented matrix $[A, B]$ to upper triangular matrix.

Step 1: Fix the first row, change 2 & 3 row with row 1

$$[A,B] \sim \begin{bmatrix} 10 & -2 & 3 & 23 \\ 0 & 10.4 & -5.6 & -37.6 \\ 0 & -3.4 & 9.1 & 34.1 \end{bmatrix} \begin{array}{l} \\ R_2 \leftrightarrow R_2 - \frac{2}{10} R_1 \\ R_3 \leftrightarrow R_3 - \frac{3}{10} R_1 \end{array}$$

Step 2: Fix 1 & 2 row, change 3 row with 2nd row.

$$\sim \begin{bmatrix} 10 & -2 & 3 & 23 \\ 0 & 10.4 & -5.6 & -37.6 \\ 0 & 0 & 7.26 & 21.80 \end{bmatrix} R_3 \leftrightarrow R_3 - \left(-\frac{3.4}{10.4}\right) R_2$$

which is an upper triangular matrix.

Step 3: Back substitution.

We get, $7.26z = 21.80 \Rightarrow z = 3$

$10.4y - 5.6z = -37.6 \Rightarrow y = -2$

$10x - 2y + 3z = 23 \Rightarrow x = 1$

checking: $10x - 2y + 3z = 23$
$10(1) - 2(-2) + 9 = 23$

Hence soln. is $x = 1,\ y = -2,\ z = 3$