Gauss Jordon Method:

This method is a modified form of Gaussian elimination method In this method, the co.eff. matrix is reduced to a diagonal matrix or unit matrix rather than a triangular matrix. Here we get the soln. without using the back substitution method.

① Using the Gauss-Jordan method solve the following equations:

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7$$

The system is equivalent to $Ax = B$.

$$\begin{pmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 13 \\ 7 \end{pmatrix}$$

Now Augmented matrix is $[A, B] = \begin{pmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{pmatrix}$

we've to reduce $[A, B]$ to diagonal matrix

Fix I row, change II, III row with row I

$$[A, B] = \begin{pmatrix} 10 & 1 & 1 & 12 \\ 0 & 9.8 & 0.8 & 10.6 \\ 0 & 0.9 & 4.9 & 5.8 \end{pmatrix} \begin{matrix} \\ R_2 \leftrightarrow R_2 - \frac{2}{10} R_1 \\ R_3 \leftrightarrow R_3 - \frac{1}{10} R_1 \end{matrix}$$

$$[A, B] \sim \begin{pmatrix} 10 & 1 & 1 & 12 \\ 0 & 9.8 & 0.8 & 10.6 \\ 0 & 0.9 & 4.9 & 5.8 \end{pmatrix}$$

Fix II, I row and change III row with row I

$$\begin{array}{c} \begin{matrix} 0 & 0.891 & 10.9 \end{matrix} \\ \sim \begin{pmatrix} 10 & 1 & 1 & 12 \\ 0 & 9.8 & 0.8 & 10.6 \\ 0 & 0 & 4.82 & 4.82 \end{pmatrix} \end{array} \begin{matrix} R_1 \leftrightarrow R_1 - \frac{1}{9.8} R_2 \\ \\ R_3 \leftrightarrow R_3 - \left(\frac{0.9}{9.8}\right) R_2 \end{matrix}$$

Fix III row, change II, I row with row III

$$\sim \begin{pmatrix} 10 & 0 & 0 & 11 \\ 0 & 9.8 & 0 & 9.8 \\ 0 & 0 & 4.82 & 4.82 \end{pmatrix} \begin{matrix} R_1 \leftrightarrow R_1 - \frac{1}{4.82} R_3 \\ R_2 \leftrightarrow R_2 - \frac{0.8}{4.82} R_3 \\ \\ \end{matrix}$$

Fix II, III row, change I row with row II

$$\sim \begin{pmatrix} 10 & 0 & 0 & 10 \\ 0 & 9.8 & 0 & 9.8 \\ 0 & 0 & 4.82 & 4.82 \end{pmatrix} \quad R_1 \leftrightarrow R_1 - \frac{1}{9.8} R_2$$

We get $10x_1 = 10 \Rightarrow x = 1$

$9.8\, y = 9.8 \Rightarrow y = 1$

$4.82\, z = 4.82 \Rightarrow z = 1$

Checking: $10x + y + z = 12$

$10(1) + 1 + 1 = 12$.