## ITERATIVE METHODS (OR) INDIRECT METHODS :

### GAUSS JACOBI (OR) JACOBI'S METHOD :

Let the system of simultaneous equations be

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right\} \quad \text{——— (1)}$$

Assume :

$|a_1| > |b_1| + |c_1|$

$|b_2| > |a_2| + |c_2|$

$|c_3| > |a_3| + |b_3|$

(The diagonal elts. should dominant, so that, the iteration process can be applied

This system of equations can also be written as :

$$\left.\begin{aligned} x &= \frac{1}{a_1}(d_1 - b_1y - c_1z) \\ y &= \frac{1}{b_2}(d_2 - a_2x - c_2z) \\ z &= \frac{1}{c_3}(d_3 - a_3x - b_3y) \end{aligned}\right\} \quad \text{——— (2)}$$

Let the first approximation be $x_0, y_0$ and $z_0$.

Sub. $x_0, y_0$, and $z_0$ in (2), we get.

$$x_1 = \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0)$$

$$y_1 = \frac{1}{b_2}(d_2 - a_2x_0 - c_2z_0)$$

$$z_1 = \frac{1}{c_3}(d_3 - a_3x_0 - b_3y_0)$$

Sub. the values of $x_1, y_1, z_1$ in (2), we get

$$x_2 = \frac{1}{a_1}(d_1 - b_1 y_1 - c_1 z_1)$$

$$y_2 = \frac{1}{b_2}(d_2 - a_2 x_1 - c_2 z_1)$$

$$z_2 = \frac{1}{c_3}(d_3 - a_3 x_1 - b_3 y_1)$$

This process is repeated till the difference btwn. two consecutive approximations is negligible.

) Solve the following system by Gauss-Jacobi method [From the Table]

$$10x - 5y - 2z = 3$$
$$4x - 10y + 3z = -3$$
$$x + 6y + 10z = -3$$

Soln:

$a_1$: $|10| > |-5| + |-2|$

$b_2$: $|-10| > |4| + |3|$

$c_3$: $|10| > |1| + |6|$

Since the diagonal elts are dominant, the iteration process is applied there.

The gn. system can be written as,

$$x = \frac{1}{10}(3 + 5y + 2z)$$

$$y = \frac{1}{10}(3 + 4x + 3z)$$

$$z = \frac{1}{10}(-3 - x - 6y)$$

I iteration: Let the initial values be

$$x_1 = \frac{1}{10}(3 + 5y_0 + 2z_0)$$

$$y_1 = \frac{1}{10}(3 + 4x_0 + 3z_0)$$

$$z_1 = \frac{1}{10}(-3 - x_0 - 6y_0)$$

Let the initial values be $x_0 = y_0 = z_0 = 0$

$$x_1 = \frac{1}{10}[3 + 5(0) + 2(0)] = 0.3$$

$$y_1 = \frac{1}{10}[3 + 4(0) + 3(0)] = 0.3$$

$$z_1 = \frac{1}{10}[-3 - 0 - 6(0)] = -0.3$$

II iteration:

$$x_2 = \frac{1}{10}(3 + 5y_1 + 2z_1)$$

$$y_2 = \frac{1}{10}(3 + 4x_1 + 3z_1)$$

$$z_2 = \frac{1}{10}(-3 - x_1 - 6y_1)$$

$x_2 = \frac{1}{10}[3 + 5(0.3) + 2(-0.3)] = 0.39$

$y_2 = \frac{1}{10}[3 + 4(0.3) + 3(-0.3)] = 0.33$

$z_2 = \frac{1}{10}[-3 - (0.3) - 6(0.3)] = -0.51$

III iteration:

$x_3 = 0.363$

$y_3 = 0.303$

$z_3 = -0.537$

IV iteration:

$x_4 = 0.3441$

$y_4 = 0.2841$

$z_4 = -0.5181$

V iteration:

$x_5 = 0.3384$

$y_5 = 0.2822$

$z_5 = -0.5048$

VI iteration:

$x_6 = 0.3401$

$y_6 = 0.2839$

$z_6 = -0.5031$

VII iteration

$x_7 = 0.3413$

$y_7 = 0.2851$

$z_7 = -0.5043$

VIII iteration:

$x_8 = 0.3416$

$y_8 = 0.2852$

$z_8 = -0.50519$

IX iteration:

$x_9 = 0.3415$

$y_9 = 0.28511$

$z_9 = -0.5052$

X iteration:

$x_{10} = 0.34148$

$y_{10} = 0.28504$

$z_{10} = -0.50522$

From 2 & 3 iteration, approximately we get

$x \approx 0.3415$

$y \approx 0.2850$

$z = -0.5052$

(2) Solve the following equations using Jacobi's iteration method:

$30x - 2y + 3z = 75$

$x + 17y - 2z = 48$

$x + y + 9z = 15$