



DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

GAUSS - SEIDEL ITERATIVE METHOD:

Let the system of simultaneous equations be

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

check: $|a_1| > |b_1| + |c_1|$

$$|b_2| > |a_2| + |c_2|$$

$$|c_3| > |a_3| + |b_3|$$

The diagonal elts. should be dominant, so that, the iteration process can be applied.

The gn. system can be written as,

$$x = \frac{1}{a_1} (d_1 - b_1y - c_1z)$$

$$y = \frac{1}{b_2} (d_2 - a_2x - c_2z)$$

$$z = \frac{1}{c_3} (d_3 - a_3x - b_3y)$$



DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

Let the 1st approximation be y_0 and $z_0 = 0$

$$x_1 = \frac{1}{a_1} (d_1 - b_1 y_0 - c_1 z_0)$$

$$y_1 = \frac{1}{b_2} (d_2 - a_2 x_1 - c_2 z_0)$$

$$z_1 = \frac{1}{c_3} (d_3 - a_3 x_1 - b_3 y_1)$$

2nd iteration:

$$x_2 = \frac{1}{a_1} (d_1 - b_1 y_1 - c_1 z_1)$$

$$y_2 = \frac{1}{b_2} (d_2 - a_2 x_2 - c_2 z_1)$$

$$z_2 = \frac{1}{c_3} (d_3 - a_3 x_2 - b_3 y_2)$$

3rd iteration:

$$x_3 = \frac{1}{a_1} (d_1 - b_1 y_2 - c_1 z_2)$$

$$y_3 = \frac{1}{b_2} (d_2 - a_2 x_3 - c_2 z_2)$$

$$z_3 = \frac{1}{c_3} (d_3 - a_3 x_3 - b_3 y_3)$$

The process is repeated until we get difference btwn. two consecutive approx. is negligible.



DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

① Solve the system of eqns:

[form the table]

$$x + y + 54z = 110$$

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

using Gauss-Seidel iteration method:

Soln: The given system is, $x + y + 54z = 110$

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

Here the diagonal elts. does not dominant, so we are interchanging the system as

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

$$x + y + 54z = 110$$

Since diagonal elts. are dominant, the iteration process is applied here. The above system can be written as

$$x = \frac{1}{27} (85 - 6y + z)$$

$$y = \frac{1}{15} (72 - 6x - 2z)$$

$$z = \frac{1}{54} (110 - x - y)$$



DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

2nd iteration:

$$x_1 = \frac{1}{27} (85 - 6y_0 + z_0)$$

$$y_1 = \frac{1}{15} (72 - 6x_1 - 2z_0)$$

$$z_1 = \frac{1}{54} (110 - x_1 - y_1)$$

Let the initial values, $y_0 = z_0 = 0$.

$$x_1 = \frac{1}{27} \times 85 = 3.1481$$

$$y_1 = \frac{1}{15} (72 - 6 \times 3.1481 - 2 \times 0) = 3.5407$$

$$z_1 = \frac{1}{54} (110 - 3.1481 - 3.5407) = 1.9131$$

3rd iteration:

$$x_2 = \frac{1}{27} (85 - 6 \times 3.5407 + 1.9131) = 2.4321$$

$$y_2 = \frac{1}{15} (72 - 6 \times 2.4321 - 2 \times 1.9131) = 3.5720$$

$$z_2 = \frac{1}{54} (110 - 2.4321 - 3.5720) = 1.9258$$

4th iteration:

$$x_3 = 2.4256$$

$$y_3 = 3.5729$$

$$z_3 = 1.9259$$



DEPARTMENT OF MATHEMATICS

UNIT – III SOLUTIONS OF EQUATIONS

IV iteration:

$$x_4 = 2.4255$$

$$y_4 = 3.5730$$

$$z_4 = 1.9259$$

V iteration:

$$x_5 = 2.4254$$

$$y_5 = 3.5730$$

$$z_5 = 1.9259$$

VI iteration:

$$x_6 = 2.4254$$

$$y_6 = 3.5730$$

$$z_6 = 1.9259$$

From I, VI iteration we get the solutions as

$$x = 2.4254 ; y = 3.5730 ; z = 1.9259$$

Q2 Solve the system of eqns. using Gauss-Seidel method:

$$x + y - 3z + 6 = 0$$

$$8x - y + z - 18 = 0$$

$$2x + 5y - 2z - 3 = 0$$

Soln:

$$x = 1.9999$$
$$y = 0.9999$$
$$z = 2.9999$$