GAUSS - SEIDEL ITERATIVE METHOD:

Let the system of simultaneous equations be

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

check: $|a_1| > |b_1| + |c_1|$

$|b_2| > |a_2| + |c_2|$

$|c_3| > |a_3| + |b_3|$

The diagonal elts. should be dominant, so that the Iteration process can be applied.

The gn. system can be written as,

$$x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$$
$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$$
$$z = \frac{1}{c_3}(d_3 - a_3x - b_3y)$$

Let the I approximation be y_0 and $z_0 = 0$

$$x_1 = \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0)$$
$$y_1 = \frac{1}{b_2}(d_2 - a_2x_1 - c_2z_0)$$
$$z_1 = \frac{1}{c_3}(d_3 - a_3x_1 - b_3y_1)$$

II iteration:

$$x_2 = \frac{1}{a_1}(d_1 - b_1 y_1 - c_1 z_1)$$

$$y_2 = \frac{1}{b_2}(d_2 - a_2 x_2 - c_2 z_1)$$

$$z_2 = \frac{1}{c_3}(d_3 - a_3 x_2 - b_3 y_2)$$

III iteration:

$$x_3 = \frac{1}{a_1}(d_1 - b_1 y_2 - c_1 z_2)$$

$$y_3 = \frac{1}{b_2}(d_2 - a_2 x_3 - c_2 z_2)$$

$$z_3 = \frac{1}{c_3}(d_3 - a_3 x_3 - b_3 y_3)$$

The process is repeated until we get difference btwn. two consecutive approx. is negligible.

(1) Solve the system of eqns:

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

[From the Table]

using Gauss-Seidel iteration method.

Soln: The given system is, $x + y + 54z = 110$
$27x + 6y - z = 85$
$6x + 15y + 2z = 72$

Here the diagonal elts. does not dominant. So we are interchanging the system as

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

Since diagonal elts. are dominant, the Iteration process is applied here. The above system can be written as

$$x = \frac{1}{27}(85 - 6y + z)$$
$$y = \frac{1}{15}(72 - 6x - 2z)$$
$$z = \frac{1}{54}(110 - x - y)$$

1 Iteration.

$$x_1 = \frac{1}{27}(85 - 6y_0 + z_0)$$
$$y_1 = \frac{1}{15}(72 - 6x_1 - 2z_0)$$
$$z_1 = \frac{1}{54}(110 - x_1 - y_1)$$

Let the Initial values, $y_0 = z_0 = 0$.

$$x_1 = \frac{1}{27} \times 85 = 3.1481$$
$$y_1 = \frac{1}{15}(72 - 6 \times 3.1481 - 2 \times 0) = 3.5407$$
$$z_1 = \frac{1}{54}(110 - 3.1481 - 3.5407) = 1.9131$$

2 Iteration.

$$x_2 = \frac{1}{27}(85 - 6 \times 3.5407 + 1.9131) = 2.4321$$
$$y_2 = \frac{1}{15}(72 - 6 \times 2.4321 - 2 \times 1.9131) = 3.5720$$
$$z_2 = \frac{1}{54}(110 - 2.4321 - 3.5720) = 1.9258$$

III iteration:

$x_3 = 2.4256$

$y_3 = 3.5729$

$z_3 = 1.9259$

IV iteration:

$x_4 = 2.4255$

$y_4 = 3.5730$

$z_4 = 1.9259$

V iteration:

$x_5 = 2.4254$

$y_5 = 3.5730$

$z_5 = 1.9259$

VI iteration:

$x_6 = 2.4254$

$y_6 = 3.5730$

$z_6 = 1.9259$

From I, VI iteration we get the solutions as

$x = 2.4254$; $y = 3.5730$; $z = 1.9259$.