1.State Lagrange's interpolation formula.

Solution:

Let y=f(x) be a function which takes the values $y_0, y_1, y_2, \ldots, y_n$ corresponding to x= $x_0, x_1, x_2, \ldots, x_n$ .

Then,Lagrange's interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots\ldots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots\ldots(x_1-x_n)}y_1 + \ldots\ldots\ldots + \frac{(x-x_0)(x-x_1)\ldots\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots\ldots(x_n-x_{n-1})}y_n$$

2.What is Inverse interpolation?

Solution:

Inverse interpolation is the process of finding the value of x corresponding to a value of y,not present in the table.

3.What is the advantage of Lagrange's formula?

Solution:Lagrange's interpolation formula can be used whether the values of x,the independent variable are equally spaced or not whether the difference of y become smaller or not.

4. Derive Newton's backward difference formula by using operator method.[Gregory Newton's backward difference interpolation formula].

Solution:

$$y(x) = f(x) = P_n(x)$$
$$= y_n + \frac{u}{1!}\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \ldots\ldots + \frac{u(u+1)(u+2)\ldots\ldots(u+(n-1))}{n!}\nabla^n y_n$$

where $u = \frac{x - x_n}{h}$

5. State Gregory Newton's forward difference interpolation formula.

Solution:

$$y(x) = f(x) = P_n(x)$$

$$= y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots\ldots\ldots$$

$$+ \frac{u(u-1)(u-2)\ldots\ldots(u-(n-1))}{n!}\Delta^n y_0$$

where $u = \frac{x - x_0}{h}$

6.State the merits and demerits of Newton's forward and backward interpolation formula.

Merits:

Newton's forward and backward interpolation formula are applicable for interpolation near the beginning and end respectively of tabulated values.

Demerits:

Newton's forward and backward interpolation formula used only for equal intervals (or) equidistant intervals.

17.Using Newton's forward difference formula,write the formula for the first,second and third order derivatives at $x = x_0$

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + ......\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + ......\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + ......\right]$$

8. Construct an Newton difference table for the points (0,-1),(1,1),(2,1) and (3,-2).

Solution:

*Difference table :*

| x | y | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 0 | -1 | | | |
| | | 1+1:2 | | |
| 1 | 1 | | 0-2:-2 | |
| | | 1-1:0 | | -3+2:-1 |
| 2 | 1 | | -3-0:-3 | |
| | | -2-1:-3 | | |
| 3 | -2 | | | |

9. State Trapezoidal rule.

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\right] \text{ where } h = \frac{b-a}{n}$$

$$= \frac{h}{2}[A + 2B]$$

where A=sum of the first and last ordinates & B=sum of the remaining ordinates.

10. Using Newton's backward difference formula,write the formula for the first,second and third order derivatives at $x = x_n$

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + ......\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + ......\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + ......\right]$$

11.State Simpson's 1/3 rule.

$$\int_{x_o}^{x_n} y\,dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})\right] \text{ where } h = \frac{b-a}{n}$$

$$= \frac{h}{3}[A + 4B + 2C]$$

where A=sum of the first and last ordinates.

B=sum of the odd ordinates

C= sum of the even ordinates