NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

1. State the disadvantages of Taylor's method.

Sol. In the differential equation dy/dx = f(x,y) , the f(x,y) function may have a complicated algebrical structure . Then the evaluation of higher order derivatives may become tedious.

2. Write down the fourth order Taylor's Algorithm.

Sol. $y_{m+1} = y_m + hy_m^1 + \left(\frac{h^2}{2!}\right) y_m^{11} + \left(\frac{h^3}{3!}\right) y_m^{111} + ..$

3. Solve $y' = x + y; \quad y(0) = 1$ by Taylor's series method. Find the values y at x = 0.1.

Solution:

Given $y' = x + y; x_0 = 0, y_0 = 1, h = 0.1$

$$y' = x + y \Rightarrow y_0{}' = x_0 + y_0 = 0 + 1 = 1$$
$$y'' = 1 + y' \Rightarrow y_0'' = 1 + y_0{}' = 1 + 1 = 2$$
$$y''' = y'' \Rightarrow y_0''' = y'' = 2$$
$$y^{iv} = y''' \Rightarrow y_0^{iv} = y''' = 2$$

$$y_1 = y_0 + \frac{h}{1!} y_0{}' + \frac{h^2}{2!} y_0{}'' + \frac{h^3}{3!} y_0{}''' + ...$$

we know that $y(0.1) = 1 + 0.1 + 0.01 + 0.0003 +$

$= 1.11033$

4. Write the Euler algorithm to find the differential equation $\frac{dy}{dx} = f(x,y)$.

Solution:

$y_{n+1} = y_n + hf(x_n, y_n)$ for the interval (x_n, y_n) when n=0,1,2,…

5. State modified Euler's algorithm to solve $y' = f(x,y), y(x_0) = y_0 \quad \text{at} \quad x = x_0 + h$

Solution:

$y_{n+1} = y_n + hf\left[x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right]$ when n=0,1,2,…

6. Using Euler's method solve $y' = x + y + xy$, $y(0) = 1$. Compute y at x=0.1 by taking h=0.05.

Solution:

$$\begin{aligned}
&\text{Given}: f(x,y) = x + y + xy \\
&x_0 = 0, y_0 = 1; h = 0.05 \\
&y_1 = y_0 + hf(x_0, y_0) \\
&\quad = 1 + 0.05[x_0 + y_0 + x_0 y_0] \\
&\quad = 1 + 0.05[0 + 1 + 0] \\
&\quad = 1.05
\end{aligned}$$

7. Compute y at x = 0.25 by modified Euler method given $y' = 2xy, y(0) = 1.$

Solution:

$$\begin{aligned}
&\text{Given} \quad y' = 2xy \\
&x_0 = 0; y_0 = 1; h = 0.25 \\
&y_1 = y_0 + hf\left[x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right] \\
&\quad = 1 + 0.25f\left[0 + \frac{0.25}{2}, 1 + \frac{0.25}{2}(2x_0 y_0)\right] \\
&\quad = 1 + 0.25f[0.25, 1] \\
&\quad = 1 + 0.25[2(0.125)(1)] \\
&\quad = 1 + 0.625 = 1.0625
\end{aligned}$$

8. Write down the Runge- Kutta method formula of second order to solve $y' = f(x,y)$ with $y(x_0) = y_0$.

Solution:

$$\begin{aligned}
&k_1 = hf(x,y) \\
&k_2 = hf\left[x + \frac{h}{2}, y + \frac{k_1}{2}\right] \\
&\text{and} \quad \Delta y = k_2 \\
&y_1 = y_0 + \Delta y
\end{aligned}$$

9. Write down the Runge kutta method formula of fourth order to solve $\frac{dy}{dx} = f(x,y)$ with $y(x_0) = y_0$

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Solution:

$k_1 = hf(x, y)$

$k_2 = hf\left[x + \frac{h}{2}, y + \frac{k_1}{2}\right]$

$k_3 = hf\left[x + \frac{h}{2}, y + \frac{k_2}{2}\right]$

$k_4 = hf\left[x + h, y + k_3\right]$

and $\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$y_1 = y_0 + \Delta y$

10. Compare Taylor's series and RK method.

Solution:

R.K methods do not require prior calculation of higher derivatives of y(x) as the Taylor method does.

Also the RK formulas involve the computation of f(x,y) at various position,instead of derivatives and this function occurs in the given equation.

11. Write Milne's predictor corrector formula.

Solution:

Milne's predictor formula

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Milne's corrector formula

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + y'_n + 2y'_{n+1}\right]$$

12. How many prior values are required to predict the next value in Milne's method?

Solution:

Four prior values

$y(x_0) = y_0; y(x_1) = y_1; y(x_2) = y_2; y(x_3) = y_3$

13. What is the error term in Milne's corrector formula?

Solution:

The error term is $-\frac{h}{90}\Delta^4 y'_0$

14. What is the error term in Milne's predictor formula?

Solution:

The error term is $\frac{14h}{45}\Delta^4 y_0'$

15. Which is better Taylor's method or R.K. Method ?

Sol. R.K methods do not require prior calculation of higher derivatives of y(x) , as the Taylor's method does . Since the differential equations using in applications are often complicated, the calculation of derivatives may be difficult.