AXIOMS OF PROBABILITY

(i) A number of telephone calls received in an office during lunch-hour has the probability function given below:

Calls (x):	0	1	2	3	4	5	6
$p(x)$:	0.05	0.20	0.25	0.20	0.15	0.10	0.05

Verify that the function is probability distribution:

Soln:

(i) $p(x) \geq 0$

(ii) $\sum_{i=0}^{6} p(x_i) = 0.05 + 0.20 + 0.25 + 0.20 + 0.15 + 0.10 + 0.05$

$= 1.0$

Thus the above two conditions are satisfied.

$\Rightarrow$ the function is probability function.

(2) Consider a random experiment of tossing a coin 3 times. Let x denote the no. of heads and y denote the no. of consecutive heads. Find

(i) probability distribution of x and y.

(ii) Distribution function of x.

(iii) probability distribution of $x+y$ and xy.

Soln:

$$S = \{HHH, HTH, HHT, THH, HTT, THT, TTH, TTT\}$$

Let x be the no. of heads.

y be the no. of Consecutive heads.

Event	HHH	THH	HTH	HHT	TTH	THT	HTT	TTT
x	3	2	2	2	1	1	1	0
y	3	2	0	2	0	0	0	0
$x+y$	6	4	2	4	1	1	1	0
xy	9	4	0	4	0	0	0	0

(i) Probability Distribution of X :

X :	0	1	2	3
$P(x)$:	$1/8$	$3/8$	$3/8$	$1/8$

Probability Distribution of Y :

Y :	0	2	3
$P(y)$:	$5/8$	$2/8$	$1/8$

(ii) Distribution function of X :

X :	0	1	2	3
$F(x)$:	$1/8$	$4/8$	$7/8$	$8/8 = 1$

$x \quad F(x) = P(X \le x)$

$0 \quad F(0) = P(X \le 0) = 1/8$

$1 \quad F(1) = P(X \le 1) = P(X=0) + P(X=1) = 1/8 + 3/8 = 4/8$

$2 \quad F(2) = P(X \le 2) = P(X=0) + P(X=1) + P(X=2) = 1/8 + 3/8 + 3/8 = 7/8$

$3 \quad F(3) = P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 1/8 + 3/8 + 3/8 + 1/8 = 1$

If A and B are independent events, prove that
(i) $\bar{A}$ and B are independent (ii) A & $\bar{B}$ are independen
(iii) $\bar{A}$ and $\bar{B}$ are independent

proof: Since A and B are independent, we have

$$p(A\cap B) = p(A) \cdot p(B) \quad \text{(defn)}$$

(i) $\bar{A}$ and B are independent

$$B = (A\cap B) \cup (\bar{A}\cap B)$$

Since $(A\cap B)$ & $(\bar{A}\cap B)$ are disjoint

$$p(B) = p(A\cap B) + p(\bar{A}\cap B)$$

$$\therefore p(\bar{A}\cap B) = p(B) - p(A\cap B)$$
$$= p(B) - p(A) \cdot p(B)$$
$$= p(B)[1 - p(A)]$$
$$= p(B) \cdot p(\bar{A})$$

$\therefore$ $\bar{A}$ and B are independent.

(ii) A and $\bar{B}$ are independent.

$$A = (A \cap B) \cup (A \cap \bar{B})$$

$$p(A) = p(A \cap B) + p(A \cap \bar{B})$$

$$\begin{aligned} p(A \cap \bar{B}) &= p(A) - p(A \cap B) \\ &= p(A) - p(A) \cdot p(B) \\ &= p(A)[1 - p(B)] \\ &= p(A)\, p(\bar{B}) \end{aligned}$$

$\therefore$ A and $\bar{B}$ are independent.

(iii) ~~we~~ $\bar{A}$ and $\bar{B}$ are independent.

we know that, $\overline{AB} = \bar{A} \cap \bar{B}$

$$\begin{aligned} p(\bar{A} \cap \bar{B}) &= p(\overline{AB}) \\ &= 1 - p(A \cup B) \\ &= 1 - [p(A) + p(B) - p(A \cap B)] \\ &= 1 - p(A) - p(B) + p(A) \cdot p(B) \\ &= [1 - p(A)]\cancel{p(B)} - p(B)[1 - p(A)] \\ &= [1 - p(B)][1 - p(A)] \\ &= \bar{A}\, \dot{B} \end{aligned}$$

$\therefore$ $\bar{A}$ & $\bar{B}$ are independent.