TOTAL PROBABILITY &BAYE'S THEOREM

(1) A problem in Mathematics is given to 3 students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. what is the prob. that the problem will be solved?

Soln: Given $P(A) = \frac{1}{2}$; $P(B) = \frac{1}{3}$; $P(C) = \frac{1}{4}$

$P(A \text{ will not solve}) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(B \text{ will not solve}) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(C \text{ will not solve}) = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore P(\text{all the three will not solve}) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$

$\therefore P(\text{all the three will solve}) = 1 - \frac{1}{4} = \frac{3}{4}$

(2) From a bag containing 4 white & 5 black balls, two ball are drawn at random. If the balls are drawn one after the other without replacement, find the prob. that,

(i) both are white ; (ii) both are black

(iii) first is white & second is black

(iv) one ball is white & the other is black.

Soln: Total no. of balls = 10 ; white ball = 4 ; Black ball = 6

(i) $P(1st\ ball\ is\ white) = \frac{4}{10}$

$P(2^{nd}\ ball\ is\ white) = \frac{3}{9}$

$\therefore P(both\ are\ white) = \frac{4}{10} \times \frac{3}{9}$

$= \frac{2}{15}$

(ii) $P(1st\ is\ black) = \frac{6}{10}$.

$P(2nd\ is\ black) = \frac{5}{9}$

$\therefore P(both\ are\ black) = \frac{6}{10} \times \frac{5}{9}$

$= \frac{1}{3}$

(iii) $P(1^{st}\ ball\ is\ white) = \frac{4}{10}$

$P(2^{nd}\ ball\ is\ black) = \frac{6}{9}$

$\therefore P(I\text{-}W \text{ and } II\text{-}B) = \frac{4}{10} \times \frac{6}{9} = \frac{4}{15}$

(iv) (a) $P(I\text{-}W \text{ and } II\text{-}B) = \frac{24}{90}$

(b) $P(I\text{-}B \text{ and } II\text{-}W) = \frac{24}{90}$

since (a) & (b) are mutually exclusive,

$P(one\ ball\ is\ white\ \&\ other\ is\ black) = \frac{24}{90} + \frac{24}{90} = \frac{48}{90} = \frac{8}{15}$

(3) Four cards are drawn without replacement. What is the prob. that they are all Aces? $\Rightarrow \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49}$

(4) A coin is tossed thrice. What is the prob. of getting all heads? $= \frac{1}{8}$

Total probability:

$$P(A) = \sum_{n=1}^{N} P(A/B_n) \cdot P(B_n)$$

Bayes Theorem:

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{\sum_{i=1}^{n} P(A_i) \cdot P(B/A_i)}$$

Problem:

In a bolt factory machines A, B, C manufacture respectively 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C.

Soln. Let A_i be the prob. of manufacturing the bolts.

Let B be the prob. of defective bolts.

$P(A_i)$	$P(B/A_i)$	$P(A_i)\,P(B/A_i)$
$P(A_1) = 0.25$	0.05	0.0125
$P(A_2) = 0.35$	0.04	0.014
$P(A_3) = 0.40$	0.02	0.008
Σ : 1.00	0.11	0.0345

WKT $P(A_i/B) = \frac{P(A_i)\,P(B/A_i)}{\sum_{i=1}^{n} P(A_i) \cdot P(B/A_i)}$

P(defective bolts manufactured by A_1 machine) $= P(A_1/B)$

$$= \frac{P(A_1)\cdot P(B/A_1)}{\sum_{i=1}^{3} P(A_1)\cdot P(B/A_i)} = \frac{0.0125}{0.0345}$$

$$= 0.3623$$

P(" A_2 machine) $= \frac{0.014}{0.0345} = 0.405$

P(A_3 machine) $= \frac{0.008}{0.0345} = 0.231$