LAGRANGIAN INTERPOLATION

Let $y = f(x)$ be a function which takes the values $y_0, y_1, y_2, \ldots, y_n$ corresponding to $x_0, x_1, x_2 \ldots x_n$.

Then Lagrangian interpolation form is

$$y = f(x)$$

$$= \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} y_1 + \ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})} y_n$$

Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for

x :	0	1	2	3	5
$f(x)$:	2	3	12		147

Soln:

$x_0 = 0$; $y_0 = 2$

$x_1 = 1$; $y_1 = 3$

$x_2 = 2$; $y_2 = 12$

$x_3 = 5$; $y_3 = 147$

By Lagrange's Interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3)$$

$$+ \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$= \frac{(x-1)(x-2)(x-5)}{-10}(2) + \frac{x(x-2)(x-5)}{4}(3)$$

$$+ \frac{x(x-1)(x-5)}{-6}(12) + \frac{x(x-1)(x-2)}{60}(147)$$

$$= x^3 + x^2 - x + 10$$

$$y = f(3) = \frac{(3-1)(3-2)(3-5)}{-10}(2) + \frac{3(3-2)(3-5)}{4}(3) +$$

$$\frac{3(3-1)(3-5)}{-6}(12) + \frac{3(3-1)(3-2)}{60}(147)$$

$$= \frac{8}{10} - \frac{18}{4} + 24 + \frac{147}{60} = 35$$

Find the missing term in the following table using Lagrange's interpolation.

x	0	1	2	3	4
y	1	3	9	-	81

Soln: 31

INVERSE INTERPOLATION:

The process of finding a value of x for the corresponding value of y is called inverse interpolation.

Inverse interpolation formula is

$$x = \frac{(y-y_1)(y-y_2)\cdots(y-y_n)}{(y_0-y_1)(y_0-y_2)\cdots(y_0-y_n)}x_0 + \frac{(y-y_0)(y-y_2)\cdots(y-y_n)}{(y_1-y_0)(y_1-y_2)\cdots(y_1-y_n)}x_1 + \cdots + \frac{(y-y_0)(y-y_1)\cdots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\cdots(y_n-y_{n-1})}x_n$$

) Find the age corresponding to the annuity value 13.6 given the table

Age (x):	30	35	40	45	50
Annuity value (y):	15.9	14.9	14.1	13.3	12.5

Soln:

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)(y-y_4)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)(y_0-y_4)} x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)(y-y_4)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)(y_1-y_4)} x_1 + \frac{(y-y_0)(y-y_1)(y-y_3)(y-y_4)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)(y_2-y_4)} x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)(y-y_4)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)(y_3-y_4)} x_3 + \frac{(y-y_0)(y-y_1)(y-y_2)(y-y_3)}{(y_4-y_0)(y_4-y_1)(y_4-y_2)(y_4-y_3)} x_4.$$

$$= \frac{(13.6-14.9)(13.6-14.1)(13.6-13.3)(13.6-12.5)}{(15.9-14.9)(15.9-14.1)(15.9-14.1)(15.9-12.5)} \times 30 +$$

$$\frac{(13.6-15.9)(13.6-14.1)(13.6-13.3)(13.6-12.5)}{(14.9-15.9)(14.9-14.1)(14.9-13.3)(14.9-12.5)} \times 35 +$$

$$\frac{(13.6-15.9)(13.6-14.9)(13.6-13.3)(13.6-12.5)}{(14.1-15.9)(14.1-14.9)(14.1-13.3)(14.1-12.5)} \times 40 +$$

$$\frac{(13.6-15.9)(13.6-14.9)(13.6-14.1)(13.6-12.5)}{(13.3-15.9)(13.3-14.9)(13.3-14.1)(13.3-12.5)} \times 45 +$$

$$\frac{(13.6-15.9)(13.6-14.9)(13.6-14.1)(13.6-13.3)}{(12.5-15.9)(12.5-14.9)(12.5-14.1)(12.5-13.3)} \times 50$$

$$= 43$$