



DEPARTMENT OF MATHEMATICS

UNIT - IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

NUMERICAL INTEGRATION BY SIMPSON'S $\frac{1}{3}$ RULE

SIMPSON'S $\frac{1}{3}$ RULE:

$$\int_{x_0}^{x_n} y \, dx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-3}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$
$$= \frac{h}{3} [A + 4B + 2C]$$

where A = Sum of the first & last ordinates

B = Sum of the odd ordinates

C = Sum of the even ordinates

ie) an even number of equal subintervals.

Example (Sin 90° = 1)
Dividing the range into 10 equal parts, find the value

of $\int_0^{\pi/2} \sin x \, dx$ by Simpson's $\frac{1}{3}$ rule



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Soln:

x	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$
$y = \sin x$	0	0.1564	0.3090	0.4540	0.5878	0.7071
x	$6\pi/20$	$7\pi/20$	$8\pi/20$	$9\pi/20$	$10\pi/20$	
$y = \sin x$	0.8090	0.8910	0.9511	0.9877	1	

By Simpson's $1/3$ rule ,

$$\int_0^{\pi/2} \sin x dx = \frac{h}{3} [(y_0 + y_{11}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8 + y_{10})]$$

$$= \frac{\pi}{20} \cdot \frac{1}{3} [(0 + 1) + 4(3.1962) + 2(2.6569)]$$

$$= 1.0000$$



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Find the value of $\log_e 5$ from $\int_0^5 \frac{dx}{4x+5}$ by Simpson's $\frac{1}{3}$ rule ($n=10$).

Soln: Here $y(x) = \frac{1}{4x+5}$

$$h = \frac{5-0}{10} = \frac{1}{2} = 0.5$$

$x :$	0	0.5	1	1.5	2	2.5	3	3.5	4
$y :$	0.2	0.1429	0.1111	0.0909	0.0769	0.0667	0.0588	0.0526	0.047
			4.5	5					
			0.0434	0.04					

By Simpson's $\frac{1}{3}$ rule,

$$\int_0^5 \frac{dx}{4x+5} = \frac{h}{3} [(y_0 + y_n) + 2(y_2 + y_4 + y_6 + \dots) + 4(y_1 + y_3 + y_5 + \dots)]$$



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$$= \frac{1}{6} [2.4148]$$

$$= 0.4025 \text{ — (1)}$$

$$\int_0^5 \frac{dx}{4x+5} = \frac{\log(4x+5)}{4} \Big|_0^5$$

$$= \frac{1}{4} (\log 25 - \log 5)$$

$$= \frac{1}{4} \log \left(\frac{25}{5} \right)$$

$$= \frac{1}{4} \log 5 \text{ — (2)}$$

From (1) & (2)

$$\Rightarrow \frac{1}{4} \log 5 = 0.4025 \quad \log e$$

$$\Rightarrow \log 5 = 1.61$$

Using Simpson's $\frac{1}{3}$ rule, evaluate

$$\int_0^{1.2} e^{-x^2} dx, \text{ taking } h=0.2.$$