
EULER METHOD:

$y_1 = y_0 + hf(x_0, y_0)$ for the interval (x_0, y_0)

$y_2 = y_1 + hf(x_1, y_1)$ " " (x_1, y_1)

. . . .

$y_{n+1} = y_n + hf(x_n, y_n)$ " " (x_n, y_n)

where $n = 0, 1, 2, \ldots$

This formula is called Euler's algorithm.

(1) Using Euler's method find $y(0.2)$ and $y(0.4)$ from $\frac{dy}{dx} = x + y$, $y(0) = 1$ with $h = 0.2$.

Soln: $\frac{dy}{dx} = f(x, y) = x + y$

Here $x_0 = 0$; $y_0 = 1$, $h = 0.2$. $x_1 = 0.2$, $y_1 = ?$

$x_2 = 0.4$ $y_2 = ?$

$y_1 = y_0 + hf(x_0, y_0)$

$= 1 + (0.2)[x_0 + y_0]$

$= 1 + (0.2)[0 + 1]$

$y(0.2) = 1.2$

$y_2 = y_1 + hf(x_1, y_1)$

$= 1.2 + (0.2)[x_1 + y_1]$

$= 1.2 + 0.2\ [0.2 + 1.2]$

$y(0.4) = 1.48$

② Using Euler's method solve $y' = x + y + xy$, $y(0) = 1$ Compute y at $x = 0.1$ by taking $h = 0.05$.

Soln:

Gn: $f(x, y) = x + y + xy$

$x_0 = 0$, $y_0 = 1$, $h = 0.05$.

$y_1 = y_0 + hf(x_0, y_0)$

$= 1 + (0.05)[x_0 + y_0 + x_0 y_0]$

$= 1 + (0.05)[0 + 1 + 0]$

$y(0.05) = 1.05$

$y_2 = y_1 + hf(x_1, y_1)$

$= 1.05 + (0.05)[x_1 + y_1 + x_1 y_1]$

$= 1.05 + (0.05)[0.05 + 1.05 + 0.05 \times 1.05]$

$y(0.1) = 1.0527$

(3) Using Euler's method find the soln. of the initial value problem $\frac{dy}{dx} = \log(x+y)$, $y(0) = 2$ at $x = 0.2$ by assuming $h = 0.2$.

Soln: $y(0.2) = 2.0602$.