RK METHOD FOR SECOND ORDER DIFFERENTIAL EQUATION

To find the soln. of $y'' = f(x, y, y')$ with the given initial cdtn. $y(x_0) = y_0$, $y'(x_0) = y_0'$.

Let $y' = z \Rightarrow \frac{dy}{dx} = z = f_1(x, y, z)$

$\Rightarrow y'' = z' \Rightarrow \frac{dz}{dx} = y'' = f_2(x, y, z)$

are simultaneous eqns

$k_1 = h f_1(x_0, y_0, z_0) = h z_0$

$l_1 = h f_2(x_0, y_0, z_0) = h y_0''$

$k_2 = h f_1\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$

$l_2 = h f_2\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$

$k_3 = h f_1\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$

$l_3 = h f_2\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$

$k_4 = h f_1\left[x_0 + h, y_0 + k_3, z_0 + l_3\right]$

$l_4 = h f_2\left[x_0 + h, y_0 + k_3, z_0 + l_3\right]$

$\Delta y = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

$\Delta z = \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$

$y_1 = y_0 + \Delta y$

$z_1 = z_0 + \Delta z$

use RK method to determine the approximate value of y at $x=0.1$ if y satisfies the DE $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$ with $y(0)=1$ and $y'(0)=0$.

soln: $y'' = 1+2xy+x^2y'$

with $y(0)=1$ and $y'(0)=0$; $h=0.1$; $x_0=0, y_0=1$; $y_0'=0$

Let $y'=z \Rightarrow \frac{dy}{dx} = z$

$y''=z' \Rightarrow \frac{dz}{dx} = y''$

$\Rightarrow y'' = 1+2xy+x^2z$

with $x_0=0, y_0=1, z_0=0$

$k_1 = hf_1(x_0, y_0, z_0)$
$= (0.1)\, z_0$
$= (0.1)0$
$= 0$

$l_1 = hf_2(x_0, y_0, z_0)$
$= (0.1)\left[1+2x_0y_0 + x_0^2z_0\right]$
$= (0.1)\left[1+2(0)+0\right]$
$= 0.1$

$k_2 = h f_1 [x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}]$

$= (0.1) f_1 [0 + \frac{0.1}{2}, 1 + \frac{0}{2}, 0 + \frac{0.1}{2}]$

$= (0.1) f_1 (0.05, 1, 0.05)$

$= 0.005$

$l_2 = h f_2 [x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}]$

$l_2 = (0.1) f_2 (0.05, 1, 0.05)$

$= (0.1) [1 + 2(0.05)(1) + (0.05)^2]$

$= 0.1100$

$k_3 = 0.0055$ $\quad$ $l_3 = 0.1100$

$k_4 = 0.0110$ $\quad$ $l_4 = 0.1202$

$\Delta y = 0.0053$ $\quad$ $\Delta z = 0.1100$

$y_1 = y_0 + \Delta y = 1.0053$ $\quad$ $z_1 = z_0 + \Delta z = 0.1100$

HW:

(1) Consider the second order initial value problem

$y'' - 2y' + 2y = e^{2t} \sin t$ with $y(0) = -0.4$ and $y'(0) = -0.6$

using fourth R.K. method, find $y(0.2)$

Soln: $y(0.2) = -0.5159$

(2) Given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$, find the value of $y(0.1)$ by using RK method of fourth order.

Soln: $y(0.1) = 0.9950$