



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

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Probability

Probability $\rightarrow$ chances of happening/occurring of an event.

We discuss about the Predictability of an event to happen/occur.

For example,

* Getting a head/tail when a coin is tossed.

* Getting a number from 1 to 6, when a die is rolled.

Mathematically, we can say that Probability of happening an event is equal to the ratio of number of favourable outcomes to number of possible outcomes.

Probability happening of an event $P = \frac{\text{No of favourable outcomes}}{\text{Total no. of possible outcomes}}$

Terms related to Probability:

Experiment:

An action where the result is uncertain even though the all possible outcomes related to it is known in advance. This is also known as random experiment.

Eg.: Throwing a die, tossing a coin etc.,

Sample Space:

Set of all possible outcomes of that experiment is an Sample space. It is denoted by S .

Eg.: If we throw a die, $S = \{1, 2, 3, 4, 5, 6\}$



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If we toss a coin, $S = \{H, T\}$.

Possible outcomes:

All possibilities related to an event are known as possible outcomes.

Tossing a coin:

When a coin is tossed, there are two possible outcomes.

Probability of getting H is $\frac{1}{2}$ or Probability of getting T is $\frac{1}{2}$.

Throwing a Die:

When a single die is thrown, there are six possible outcomes

1, 2, 3, 4, 5 and 6.

Probability of getting any one of no is $\frac{1}{6}$.

Example: 1

There are 5 marbles in a bag, 3 of them are red and 2 of them are blue. What is the probability that a blue marble will be picked?

Sol:

Number of favourable outcomes = 2 ($\because$ 2 blue marbles)

Total number of outcomes = 5 (because 5 marbles in total)

$$\text{Probability} = \frac{2}{5} = 0.4$$

Event:

Event is the single result of an experiment.

Eg: Getting a head is an event related to tossing of a coin.



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Sol: Bag contains 20 black marbles and there is no red marble in the bag.

So favourable cases = 0

Total outcomes = 20

Required Probability = $\frac{0}{20} = 0$.

Equally Likely Events :

Equally likely events $\Rightarrow$ Events related to an experiment.

Probability of occurrence of each event is same.

Eg :: When a dice is rolled the possible outcome of getting an odd number = Possible outcome of getting an even number = 3.

So, getting a even number/odd number are equally likely events.

Complement of an Event :

Set of all outcomes in the sample space that are not included in the outcomes of event A.

Complement of event A is represented by $\bar{A}$.

Probability of complement of an event is

$$P(\bar{A}) = 1 - P(A).$$

Example: 4 A single card is chosen at random from a standard deck of 52 playing cards. what is the Probability of choosing a card that is not a King?



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Types of Events:-

1. Certain event

2. Impossible event

Certain event :-

Certain to occur.

ie, S (Sample space) is a certain event.

Probability of certain event is 1.

ie, $P(S) = 1$.

Impossible event :-

No chance of occurring.

ie, ϕ is an impossible event.

Probability of impossible event is 0.

ie, $P(\phi) = 0$.

Example: 2 A teacher chooses a student at random from a class of 30 boys. What is the Probability that the student chosen is a boy?

Sol: All the students are boys, so chosen may be any one.

ie, Favourable cases = Total Cases = 30

$$\text{Probability} = \frac{30}{30} = 1$$

Example: 3 A bag contains 20 black marbles, if a marble is picked at random from the bag. Find the Probability that marble picked is of red colour.



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Sol: A standard deck contains 4 King.

Probability of getting a King = $\frac{4}{52}$.

Probability of not getting a King = $1 - \text{Probability of getting a King}$

$$= 1 - \frac{4}{52}$$

$$= \frac{52-4}{52}$$

$$= \frac{48}{52}$$

$$= \frac{12}{13}$$

Mutually Exclusive and Exhaustive Events:

Mutually exclusive $\Rightarrow$ Two events E_1 and E_2 related to an experiment E , having sample space S are mutually exclusive. Probability of occurrence of both events simultaneously is zero.

ie) $P(E_1 \cap E_2) = 0$.

Eg: When a coin is tossed either head or tail will appear.

Head and tail can not occur simultaneously then occurrence of a head or a tail are two mutually exclusive events.

Mutually exhaustive $\Rightarrow$ Two events E_1 and E_2 related to an experiment E , having sample space S are mutually exhaustive

if probability of occurrence of event E_1 or E_2 is 1.

ie) $P(E_1 \cup E_2) = 1$.



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Eg: Let A be Probability of getting an even number when a dice is rolled and B be the Probability of getting an odd number. The Probability of occurrence of event A or event B is 1. i.e., Any of the even can occur so they are mutually exhaustive.

Note:

Events $E_1, E_2, E_3, \dots, E_n$ related to S

1. Mutually exclusive if

$$P(E_1 \cap E_2 \cap E_3 \dots \cap E_n) = 0 \text{ or } E_1 \cap E_2 \cap E_3 \dots \cap E_n = \emptyset.$$

2. Mutually exhaustive if

$$P(E_1 \cup E_2 \cup E_3 \dots \cup E_n) = 1 \text{ or } E_1 \cup E_2 \cup \dots \cup E_n = S.$$

Dependent events :-

Two events are dependent if the outcome or occurrence of the first affects the outcome or occurrence of the second, so the Probability is changed.

Independent events :-

Two events A and B are independent if occurring or non-occurring of A does not affect the occurring or non-occurring of B.

If A and B are independent events, then

$$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B)$$

Eg: Getting head after tossing a coin and getting a 5 on a rolling single 6-sided die are independent events.



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In general, we say if events $E_1, E_2, \dots, E_n$ related to an experiment are independent, then

$$P(E_1 \cap E_2 \cap E_3 \dots E_n) = P(E_1)P(E_2)P(E_3) \dots P(E_n)$$

$$(or) P(E_1 \cup E_2 \cup E_3 \dots E_n) = 1 - P(\bar{E}_1)P(\bar{E}_2) \dots P(\bar{E}_n)$$

Example: 5

A coin is chosen at random from a standard deck of 52 playing cards. without replacing it, a second card is chosen. What is the Probability that the first card chosen is a queen and the second card chosen is a jack?

$$sol: P(\text{queen on first pick}) = \frac{4}{52}$$

$$P(\text{jack on 2nd pick given queen on 1st pick}) = \frac{4}{51}$$

why 51, because one card is already picked)

$$P(\text{queen and jack}) = \frac{4}{52} \times \frac{4}{51} = \frac{4}{663}$$

Example: 6

A coin is tossed and a single 6-sided die is rolled. Find the Probability of getting the head side of the coin and getting a 3 on the die.

$$sol: \therefore \text{Probability of getting a head when a coin is tossed} = \frac{1}{2}$$

$$\text{Probability of getting a 3 when a die is rolled} = \frac{1}{6}$$

$$\text{Now, the required Probability that both occurs at the same time} = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$



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Rule's/Theorems Related to Probability :

1 Addition Rule of Probability :

When two events A and B are mutually exclusive, the Probability that A or B will occur, is the sum of the Probability of each event.

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B)$$

But when two events A and B are non-mutually exclusive, the Probability that A or B will occur, is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication Theorem of Probability :

When two events A and B are mutually exclusive, the Probability that A and B will occur simultaneously is given as

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B) = P(A) \cdot P(B) \quad (\text{A and B are independent event})$$

Example: From a well-shuffled pack of 52 cards, a card is drawn at random, find the Probability that it is either a heart or a queen.

Sol: A = Probability of getting a heart card

B = Probability of getting a queen card



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$$P(A) = \frac{13}{52} \quad P(B) = \frac{4}{52} \quad P(A \cap B) = \frac{1}{52} \quad (\text{one heart card is a queen})$$

$$\text{Required Probability} = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{13+4-1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}$$

Example: 8 Eight persons A, B, C, D, E, F, G, and H appeared for an interview. Find the Probability that both A and D are Selected in the interview?

Sol: Probability that A is Selected $P(A) = \frac{1}{8}$

Probability that D is Selected $P(D) = \frac{1}{8}$

Probability that both are selected $= \frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$

Law of Total Probability:

If $E_1, E_2, E_3, \dots, E_n$ be n mutually exclusive events related to an experiment, then Probability of an event A which occurs with E_1 or E_2 or $E_3, \dots, E_n$ is given by

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \dots + P(E_n)P(A/E_n).$$

Conditional Probability:

The Conditional Probability of an event B in relation to an event A is the Probability of that event B occurs given that event A has already been occurred.



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The Notation for Conditional Probability is $P(B/A)$.

$$1. \quad P(A/B) = \frac{P(A \cap B)}{P(B)} \quad \text{and} \quad P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Example: 9 A Mathematics teacher conducted two tests in his class. 25% of the students passed both tests and 42% of the students passed the first test. What percent of the students passed the second test given that they have already passed the first test?

$$\text{Sol: } P(B/A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(\text{Second/first}) = \frac{P(\text{first and second})}{P(\text{first})} = \frac{0.25}{0.42} \approx 0.60 = 60\%$$

Types of Questions:

1. Based on Coins:

Based on tossing of coins (s) and obtaining a Particular face (H/T) or obtaining same face on two or more coins.

Example: 10 what is the Probability of each outcome, when a coin is tossed?

$$\text{Sol: } S = \{H, T\} \quad \text{ie., } n(s) = 2$$

$$P(\text{Head}) = \frac{1}{2}$$

$$P(\text{Tail}) = \frac{1}{2}$$



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Example: 11

A coin is tossed twice, then find the Probability that a head is obtained atleast once.

Sol: When a coin is tossed twice, then possible outcome

$$n(S) = \{HH, HT, TH, TT\} = 4.$$

A head is obtained atleast once

$$n(P) = \{HH, HT, TH\} = 3.$$

$$\text{Probability} = \frac{n(P)}{n(S)} = \frac{3}{4}.$$

2. Based on Dice:

Based on rolling of one or more dice and getting a Particular number on the face or a Particular sum on faces of the dice etc..

Example 12: A single 6-sided die is rolled. What is the Probability of getting an even number and getting an odd number?

Sol: Possible outcomes are 1, 2, 3, 4, 5 and 6.

Even numbers are 2, 4 and 6.

ie, 3 Outcomes.

So, the number of favourable (even) outcomes = 3

Total no of outcomes = 6

$$\text{Probability of getting an even number} = \frac{3}{6} = \frac{1}{2}$$

$$\text{Probability of getting an odd number} = \frac{3}{6} = \frac{1}{2}$$



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Example: 13

A single 6-sided die is rolled. What is the probability of getting a 2 or a 5?

• Total outcomes when a die is rolled = 6

• Probability of getting a number in a single throw of die $\} = \frac{1}{6}$.

$$P(2) = \frac{1}{6} \text{ and } P(5) = \frac{1}{6}$$

Probability of getting either 2 or 5

$$= P(2 \text{ or } 5)$$

$$= P(2) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

3. Based on Playing Cards:

Total no of cards = 52.

i) So there are 13 cards of each suit clubs, Diamonds, Hearts and spades.

ii) There are 4 Aces, 4 Jacks, 4 Queens and 4 Kings.

iii) There are 26 red and 26 black cards.

iv) There are 12 face cards.

Example: 14 A total of five cards are chosen at random from a standard deck of 52 playing cards. What is the probability of choosing 5 aces?



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sol: There are only 4 aces, so we cannot choose 5 aces.

It is impossible event.

$$\text{Probability (5 aces)} = \frac{0}{52} = 0.$$

Example: 15 A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing a king or a club?

sol: There are 4 King in a standard deck and 13 club card.

Also, 1 King is of club. So Probability of getting a King = $\frac{4}{52}$

$$\text{Probability of getting a club} = \frac{13}{52}$$

$$\text{Probability of getting a King of club} = \frac{1}{52}$$

$$\begin{aligned} \text{Probability of getting a king or club} &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\ &= \frac{4+13-1}{52} = \frac{16}{52} = \frac{4}{13}. \end{aligned}$$

4. Based on Marbles or Balls:

Questions are based on choosing a ball or a marble of Particular colour from one or more bag containing different Coloured balls or marbles.

Example: 16 A glass jar contains 1 red, 3 green, 2 blue and 4 yellow marbles. If a single marble is chosen at random from the jar, what is the probability that it is yellow or green?



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Sol: Total marble = $1 + 3 + 2 + 4 = 10$

$$n(s) = 10$$

$$\text{Probability of getting a yellow marble} = \frac{4}{10}$$

$$\text{Probability of getting a green marble} = \frac{3}{10}$$

Since, the events are mutually exclusive.

$$\begin{aligned} P(\text{yellow or green}) &= P(\text{yellow}) + P(\text{green}) \\ &= \frac{4}{10} + \frac{3}{10} = \frac{7}{10} \end{aligned}$$

Example: 17 A person has 2 bags. He has 3 black and 4 white balls in one bag and 4 black and 3 white balls in another bag. Find the Probability of getting a black ball.

Sol: 1st bag = 3 black 4 white balls

2nd bag = 4 black 3 white balls

Case: I If 1st bag was chosen among the two bags and ball drawn is black, then

$$\text{Probability} = \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$$

$\left[\frac{1}{2} \right]$ represents
Probability of
choosing a bag
out of two

Case: II If 2nd bag was chosen among the two bags and ball drawn is black, then

$$\text{Probability} = \frac{1}{2} \times \frac{4}{7} = \frac{4}{14}$$



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Example: 19

A person can hit a target 4 out of 7 shots. If he fires 10 shots, what is the probability that he hit the target

twice?

Sol: $n = 10, r = 2$

Success $\Rightarrow P = \frac{4}{7}$

Failure $\Rightarrow q = 1 - \frac{4}{7} = \frac{3}{7}$

$$\begin{aligned}
 P(\text{hit the target twice}) &= {}^{10}C_2 \left(\frac{4}{7}\right)^2 \left(\frac{3}{7}\right)^8 \\
 &= \frac{10!}{2!(10-2)!} \left(\frac{4}{7}\right)^2 \left(\frac{3}{7}\right)^8 \\
 &= 45 \frac{4^2 3^8}{7^{10}}
 \end{aligned}$$

Example: 20

The Probability that it is Friday and a student is absent is 0.03. Since there are 5 school days in a week, the Probability that it is Friday is 0.2. What is the Probability that a student is absent, given that today is Friday?

Sol: Probability of knowing that it is Friday and a student is absent = 0.03

Probability of day be a Friday = 0.2

If it is given that today is Friday then, it is a Conditional Probability.

$$P(\text{absent} | \text{Friday}) = \frac{P(\text{Friday and absent})}{P(\text{Friday})} = \frac{0.03}{0.2}$$



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$$\begin{aligned}
 \text{Probability of choosing the } \left. \begin{array}{l} \text{black ball among the} \\ \text{two bag} \end{array} \right\} &= P(1^{\text{st}} \text{ bag}) \text{ or } P(2^{\text{nd}} \text{ bag}) \\
 &= \frac{3}{14} + \frac{4}{14} \\
 &= \frac{3+4}{14} \\
 &= \frac{7}{14} \\
 &= \frac{1}{2}
 \end{aligned}$$

5. Miscellaneous (choosing a student, hitting a target, etc.)

Example: 18

In a Mathematics class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the Probability of choosing a girl or an 'A' grade student?

Sol: Total boys = 17 Total girls = 13
Girls getting A grade = 5 Boys getting A grade = 4

Probability of choosing a girl = $\frac{13}{30}$

Probability of choosing a A grade student = $\frac{9}{30}$

Now, A grade student chosen can be a girl,

Probability of choosing it = $\frac{5}{30}$

Probability of choosing a girl or a A grade student
= $\frac{13}{30} + \frac{9}{30} - \frac{5}{30} = \frac{22-5}{30} = \frac{17}{30}$