DERIVATIVES FROM DIFFERENCE TABLES – DIVIDED DIFFERENCE AND FINITE DIFFERENCES :

The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds. Find the initial acceleration using the entire data

Time (sec) :	0	5	10	15	20
Velocity (m/sec) :	0	3	14	69	228.

Soln: Velocity $\rightarrow v$

Acceleration $\rightarrow \frac{dv}{dt}$

Initial acceleration $= \frac{dv}{dt}$ at $t=0$

t	v	Δv	$\Delta^2 v$	$\Delta^3 v$	$\Delta^4 v$
0	0				
		3			
5	3		8		
		11		36	
10	14		44		24
		55		60	
15	69		104		
		159			
20	228				

Initial acceleration $\frac{dv}{dt}$ at $t=0$ is

$$= \frac{1}{h}\left[\Delta v_0 - \frac{1}{2}\Delta^2 v_0 + \frac{1}{3}\Delta^3 v_0 - \frac{1}{4}\Delta^4 v_0 + \cdots\right]$$

$h=5$

$$= \frac{1}{5}\left[3 - \frac{1}{2}\times 8 + \frac{1}{3}\times 36 - \frac{1}{4}\times 24\right]$$

$$= \frac{1}{5}\left[3-4+12-6\right]$$

$$= \frac{1}{5}\left[5\right]$$

$$= 1$$

(5) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=51$ from the following data :

x	50	60	70	80	90
y	19.96	36.65	58.81	77.21	94.61

Soln: Here $h=10$, $x_0=50$, $x=51$

$$\therefore u = \frac{x-x_0}{h} = \frac{51-50}{10} = 0.1$$

$$\left(\frac{dy}{dx}\right)_{x=51} = \left(\frac{dy}{dx}\right)_{u=0.1} = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$u = \frac{x - x_0}{10}$, $x_0 = 50$

x	u	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
50	0	19.96				
			16.69			
60	1	36.65		5.47		
			22.16		-9.23	
70	2	58.81		-3.76		11.99
			18.4		2.76	
80	3	77.21		-1.00		
			17.4			
90	4	94.61				

$$\left(\frac{dy}{dx}\right)_{u=0.1} = \frac{1}{10}\left[16.69 + \frac{2\times 0.1 - 1}{2}\times 5.47 + \frac{3\times 0.1^2 - 6\times 0.1 + 2}{6}(-9.23) + \frac{4\times 0.1^3 - 18\times 0.1^2 + 22\times 0.1 - 6}{24}\times 11.99\right]$$

$= \frac{1}{10}[16.69 - 2.188 - 2.1998 - 1.9863]$

$= 1.03159$

(2) Find the first, second & third derivatives of $f(x)$ at $x = 1.5$ if

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.00	13.625	24.0	38.875	59.0

Soln: $f'(1.5) = 4.75$; $f''(1.5) = 9$; $f'''(1.5) = 6$