



DEPARTMENT OF MATHEMATICS

UNIT - IV INTERPOLATION , NUMERICAL DIFFERENTIATION & INTEGRATION

NUMERICAL INTEGRATION BY TRAPEZOIDAL

TRAPEZOIDAL RULE :

$$\int_{x_0}^{x_n} y \, dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$$
$$= \frac{h}{2} [A + 2B]$$

where A = sum of the first & last ordinates

B = sum of the remaining ordinates.

Q) using trapezoidal rule, evaluate $\int_{-1}^1 \frac{dx}{1+x^2}$ taking 8 intervals.

Soln: Given $y(x) = \frac{1}{1+x^2}$

Here $h = \frac{b-a}{n}$ where $a = -1$, $b = 1$, and $n = 8$

$$\Rightarrow h = \frac{2}{8} = 0.25$$



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$$\begin{array}{cccccccccc} x : & -1 & -0.75 & -0.5 & -0.25 & 0 & 0.25 & 0.5 & 0.75 & 1 \\ y : & 0.5 & 0.64 & 0.8 & 0.9412 & 1 & 0.9412 & 0.8 & 0.64 & 0.5 \end{array}$$

Trapezoidal rule,

$$\begin{aligned} \int_{-1}^1 \frac{1}{1+x^2} dx &= \frac{h}{2} \left[(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \right] \\ &= \frac{h}{2} \left[\text{sum of the first and last ordinates} \right. \\ &\quad \left. + 2 \times \text{sum of the remaining ordinates} \right] \\ &= \frac{0.25}{2} \left[(0.5 + 0.5) + 2(0.64 + 0.8 + 0.9412 + \right. \\ &\quad \left. 0.9412 + 0.8 + 0.64) \right] \\ &= \frac{0.25}{2} \times 12.5248 \\ &= 1.5656 \end{aligned}$$

② Dividing the range into 10 equal parts, find the value

$$\int_0^{\pi/2} \sin x dx \text{ by (i) Trapezoidal rule}$$



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Soln: $x : 0, \frac{\pi}{20}, \frac{2\pi}{20}, \frac{3\pi}{20}, \frac{4\pi}{20}, \frac{5\pi}{20}$

$y = \sin x : 0, 0.1564, 0.3090, 0.4540, 0.5878, 0.7071$

$x : \frac{6\pi}{20}, \frac{7\pi}{20}, \frac{8\pi}{20}, \frac{9\pi}{20}, \frac{10\pi}{20}$

$y = \sin x : 0.8090, 0.8910, 0.9511, 0.9877, 1$

By Trapezoidal rule;

$$\int_0^{\pi/2} \sin x dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{10})]$$

Where $\boxed{h = \frac{\pi}{2} - 0}{10} = \frac{\pi}{20}$

$$= \frac{\pi}{2} [(0+1) + 2(0.1564 + 0.3090 + 0.4540 + 0.5878 + 0.7071 + 0.8090 + 0.8910 + 0.9511 + 0.9877)]$$
$$= \frac{\pi}{20} \cdot \frac{1}{2} [12.7062]$$
$$= 0.9980$$