NUMERICAL INTEGRATION BY TRAPEZOIDAL

TRAPEZOIDAL RULE :

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$$

$$= \frac{h}{2}\left[A+2B\right]$$

where A = Sum of the first & last ordinates

B = Sum of the remaining ordinates.

(1) Using trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

Soln: Gn: $y(x) = \frac{1}{1+x^2}$

Here $h = \frac{b-a}{n}$ where $a=-1$, $b=1$, and $n=8$

$\Rightarrow h = \frac{2}{8} = 0.25$

x:	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
y:	0.5	0.64	0.8	0.9412	1	0.9412	0.8	0.64	0.5

Trapezoidal rule,

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\dots+y_{n-1})\right]$$

$$= \frac{h}{2}\,[\text{sum of the first and last ordinate} + 2\times\text{sum of the remaining ordinates}$$

$$= \frac{0.25}{2}\left[(0.5+0.5)+2(0.64+0.8+0.9412+0.9412+0.8+0.64)\right]$$

$$= \frac{0.25}{2}\times 12.5248$$

$$= 1.5656$$

(2) Dividing the range into 10 equal parts, find the value of $\int_0^{\pi/2} \sin x\,dx$ by (i) Trapezoidal rule

Soln:

x :	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$
$y = \sin x$:	0	0.1564	0.3090	0.4540	0.5878	0.7071
x :	$6\pi/20$	$7\pi/20$	$8\pi/20$	$9\pi/20$	$10\pi/20$	
$y : \sin x$:	0.8090	0.8910	0.9511	0.9877	1	

By Trapezoidal rule;

$$\int_0^{\pi/2} \sin x \, dx = \frac{h}{2}\left[(y_0 + y_{10}) + 2(y_1 + y_2 + \cdots + y_{10})\right]$$

Here $\boxed{h = \dfrac{\frac{\pi}{2} - 0}{10} = \dfrac{\pi}{20}}$

$$= \frac{h}{2}\left[(0+1) + 2(0.1564 + 0.3090 + 0.4540 + 0.5878 + 0.7071 + 0.8090 + 0.8910 + 0.9511 + 0.9877)\right]$$

$$= \frac{\pi}{20} \cdot \frac{1}{2}\left[12.7062\right]$$

$$= 0.9980$$